Assignment Sheet 8

Assignment 28  Fuzzy Relational Equations

Let \( X = \{x_1, x_2, x_3\} \) and \( Y = \{y_1, y_2, y_3, y_4\} \) be two sets and \( \mu : X \rightarrow [0,1] \) and \( \nu : Y \rightarrow [0,1] \) two fuzzy sets on \( X \) and \( Y \), respectively, which are defined as follows:

\[
\begin{align*}
\mu(x_1) &= 0.1, & \mu(x_2) &= 0.7, & \mu(x_3) &= 1.0, \\
\nu(y_1) &= 0.4, & \nu(y_2) &= 1.0, & \nu(y_3) &= 0.8, & \nu(y_4) &= 0.3.
\end{align*}
\]

a) How can you find out whether the relational equation \( \mu \circ \varrho = \nu \) has a solution, i.e. whether there is a fuzzy relation \( \varrho \) that satisfies this equation?

b) If the relational equation \( \mu \circ \varrho = \nu \) has a solution, determine a solution. Are there other solutions than the one you found?

Assignment 29  Fuzzy Relational Equations

Let \( X = \{x_1, x_2, x_3\} \) and \( Y = \{y_1, y_2\} \) be two sets. Consider the fuzzy sets \( \mu_1, \mu_2, \mu_3 \) on \( X \) and \( \nu_1, \nu_2, \nu_3 \) on \( Y \) which are defined as shown in the two tables below.

<table>
<thead>
<tr>
<th></th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( y_1 )</th>
<th>( y_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_1 )</td>
<td>1.0</td>
<td>0.6</td>
<td>0.2</td>
<td>1.0</td>
<td>0.4</td>
</tr>
<tr>
<td>( \mu_2 )</td>
<td>0.0</td>
<td>0.8</td>
<td>1.0</td>
<td>0.6</td>
<td>1.0</td>
</tr>
<tr>
<td>( \mu_3 )</td>
<td>0.9</td>
<td>0.1</td>
<td>0.0</td>
<td>0.9</td>
<td>0.5</td>
</tr>
</tbody>
</table>

a) Show that the system consisting of the two relational equations \( \mu_1 \circ \varrho = \nu_1 \) and \( \mu_2 \circ \varrho = \nu_2 \) has a solution. Find the greatest solution of this system.

b) Is the fuzzy relation that can be computed as the union (maximum) of the two Cartesian products \( \mu_1 \otimes \nu_1 \) and \( \mu_2 \otimes \nu_2 \) also a solution of the system of relational equations considered in a)?

c) Show that the system consisting of the three relational equations \( \mu_i \circ \varrho = \nu_i \), \( i = 1, 2, 3 \), does not have any solution.
Assignment 30  Fuzzy Control based on Relational Equations

Let $X = \{1, 2, 3\}$ and $Y = \{10, 20, 30\}$ be two sets, $\mu_1, \mu_2$ fuzzy sets on $X$, and $\nu_1, \nu_2$ fuzzy sets on $Y$, which are defined as shown in the two tables below.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1$</td>
<td>0.0</td>
<td>0.5</td>
<td>1.0</td>
<td>$\nu_1$</td>
<td>0.0</td>
<td>0.6</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>1.0</td>
<td>0.4</td>
<td>0.0</td>
<td>$\nu_2$</td>
<td>1.0</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Consider a fuzzy controller with the following rule base:

if $x$ is $\mu_1$ then $y$ is $\nu_1$,
if $x$ is $\mu_2$ then $y$ is $\nu_2$.

Use the Gödel relation to determine the fuzzy output of this controller for the fuzzy input $(1 : 0.1, 2 : 1, 3 : 0)$. 