Assignment Sheet 7

Assignment 24 Fuzzy Set Operations

Let the following two fuzzy sets be given:

\[\begin{align*}
\mu_1(x) &= \begin{cases} 
20 - x & \text{if } x < 20 \\
0 & \text{otherwise}
\end{cases} \\
\mu_2(x) &= 0.95^x
\end{align*}\]

Compute and draw

a) the intersection of \(\mu_1\) and \(\mu_2\) using the algebraic product \(\mathcal{T}_{\text{prod}}\),

b) the union of \(\mu_1\) and \(\mu_2\) using the algebraic sum \(\mathcal{L}_{\text{sum}}\),

c) the degree to which \(\mu_1\) is a subset of \(\mu_2\) using the Łukasiewicz implication.

Assignment 25 Set Representation

Use the fuzzy sets \(\mu_1\) and \(\mu_2\) given in assignment 24. Perform the following computations in the alpha cut representations of \(\mu_1\) and \(\mu_2\), draw the resulting fuzzy sets and give the explicit (vertical) representation:

a) \(z = x + 2 \cdot y\)

b) \(z = \frac{x \cdot y}{4}\).

Assignment 26 Quantifiers

To describe the concept “\(x\) is a small number”, let \(x \in \mathbb{N} \cup \{0\}\) and two membership functions \(\mu_1(x)\) and \(\mu_2(x)\) be defined as follows:

\[\begin{align*}
\mu_1(x) &= \begin{cases} 
\frac{20 - x}{20} & \text{if } x < 20 \\
0 & \text{otherwise}
\end{cases} \\
\mu_2(x) &= 0.95^x
\end{align*}\]

Compute the fuzzy truth value of the proposition “All single-digit numbers of the decimal number system are small” with respect to both \(\mu_1\) and \(\mu_2\). As conjunction, utilize both

a) \(\mathcal{T}_{\min}(a, b) = \min\{a, b\}\),

b) \(\mathcal{T}_{\text{prod}}(a, b) = a \cdot b\).

Are the results plausible with regard to your intuition?
Assignment 27 Linguistic Variables

Driven by the discussions about possible health consequences of excessive consumption of sweets, Saint Nicholas (the German “Nikolaus”) wants to add another criterion to his well known one of “clean shoes”, namely the nutritional condition of his clientele. He uses all criteria for purpose of computing the amount of sweets in each case.

In the last days he spent lots of his time in the revision of his rules of the thumb used for this purpose. At the same time he came up with the idea of being assisted by a new Japanese “Robopet” for the future.

In its enclosed manual he got to know that Robopet works with fuzzy logic. In order to implement his rules that are written in natural language, Saint Nicholas now needs linguistic variables for the following variables:

- cleanliness of the shoes,
- degree of over weightiness,
- amount of sweets.

Find the linguistic variables with which one can describe these concepts. Note that in his rules Saint Nicholas wants to utilize both expressions (such as “great many more than 5 sweets” or “somewhat clean shoes”) and logical operations (as found in “slightly overweight or up to 130% of the normal weight”).

Hint: For the basic set of the concept “over weightiness”, it might come in handy to use, e.g. the ration of actual weight and recommended maximum weight.