Fuzzy Systems
Prof. Dr. Rudolf Kruse, Christoph Doell

Assignment Sheet 6

Assignment 20   **Fuzzy Relations**

Let the fuzzy relation R be defined on the sets $X_1 = \{a, b, c\}$, $X_2 = \{s, t\}$, $X_3 = \{x, y\}$ and $X_4 = \{i, j\}$. Furthermore, let R be different than 0 at the following positions:

$$R(a, t, y, j) = 0.2,$$
$$R(b, s, x, j) = 0.5,$$
$$R(a, s, y, j) = 1.0,$$
$$R(a, s, y, i) = 0.9,$$
$$R(b, t, y, i) = 0.7,$$
$$R(c, s, y, j) = 0.3.$$

a) Compute the following projections of R:

$$R_{1,2,4} = [R \downarrow \{X_1, X_2, X_4\}],$$
$$R_{1,3} = [R \downarrow \{X_1, X_3\}],$$
$$R_4 = [R \downarrow \{X_4\}].$$

b) Compute the following cylindric extensions:

$$[R_{1,2,4} \uparrow \{X_3\}],$$
$$[R_{1,3} \uparrow \{X_2, X_4\}],$$
$$[R_4 \uparrow \{X_1, X_2, X_3\}].$$

Assignment 21   **Fuzzy Relations**

Prove that not every fuzzy relation R on $X \times Y$ is the Cartesian product of two fuzzy sets A of X and B of Y.

Assignment 22   **Fuzzy Relations**

Let R be a fuzzy relation on $X \times Y$ and S, T fuzzy relations on $Y \times Z$. Find an example where $R \circ (S \cap T) \subset (R \circ S) \cap (R \circ T)$ holds.

Assignment 23   Fuzzy Binary Relations

The fuzzy binary relation R is defined on set $X = \{1, 2, \ldots, 100\}$ and $Y = \{50, 51, \ldots, 100\}$ and represents the relation "x is much smaller than y". It is defined by its membership function

$$R(x, y) = \begin{cases} 1 - \frac{x}{y}, & \text{if } x \leq y \\ 0, & \text{otherwise,} \end{cases}$$

whereas $x \in X$ and $y \in Y$.

a) What is the domain of R?

b) What is the range of R?

c) What is the height of R?

d) Calculate R^{-1}.