Assignment Sheet 4

Assignment 12  Fuzzy Set Operations

Let the following two fuzzy sets be given:

\[ \mu_1, \mu_2 \]

Compute and draw for each of the pairs:

a) the complement of \( \mu_1 \) w.r.t. \( U = [1, 8] \) using the standard fuzzy negation,

b) the intersection of \( \mu_1 \) and \( \mu_2 \) using the standard fuzzy t-norm \( \min \),

c) the intersection of \( \mu_1 \) and \( \mu_2 \) using the algebraic product \( \prod \),

d) the intersection of \( \mu_1 \) and \( \mu_2 \) using the Łukasiewicz t-norm \( \ Łuka \),

e) the union of \( \mu_1 \) and \( \mu_2 \) using the standard fuzzy t-conorm \( \max \),

f) the union of \( \mu_1 \) and \( \mu_2 \) using the algebraic sum \( \sum \),

g) the union of \( \mu_1 \) and \( \mu_2 \) using the Łukasiewicz t-conorm \( \ Łuka \).
Now, consider the class of increasing generator functions
\[ g_\lambda(a) = \frac{a}{\lambda + (1 - \lambda)a}. \]

Apply the given theorem, which allows to construct an involutive fuzzy negation from an arbitrary continuous and strictly increasing function \( g \) with \( g(0) = 0 \). Draw the resulting fuzzy negation for several values of \( \lambda \).

**Assignment 14  Greatest t-norm**

Motivate graphically that the Minimum is the greatest \( t \)-norm.

Draw a 3D-Plot for two fuzzy truth variables in \([0,1]\) and the corresponding output variable in \([0,1]\) as e.g. done on slide 8 of the lecture on fuzzy logic.

Start drawing the values necessary for fulfilling the crisp logic, then iteratively add the properties of \( t \)-norms and their graphical meanings in your drawing.

**Assignment 15  Fuzzy Conjunction**

Prove the following theorem which was given in the lecture:

**Theorem:** For all \( t \)-norms \( \top \) and all fuzzy truth values \( a, b \in [0,1] \) it is
\[ \top_{-1}(a, b) \leq \top(a, b) \leq \top_{\text{min}}(a, b), \]

where \( \top_{\text{min}}(a, b) = \min\{a, b\} \) is the standard fuzzy conjunction and \( \top_{-1} \) is the so-called drastic product
\[ \top_{-1}(a, b) = \begin{cases} a & \text{if } b = 1, \\ b & \text{if } a = 1, \\ 0 & \text{otherwise.} \end{cases} \]