Fuzzy Systems
Prof. Dr. Rudolf Kruse, Christoph Doell

Assignment Sheet 2

Assignment 5 Lattices/Boolean Algebras

The transfer from logic to set theory is possible because both systems have basically the same structure. This structure is captured by the algebraic notion of a Boolean algebra. A Boolean algebra on a set B is defined as quadruple $\mathcal{B} = (B, +, \cdot, \bar{})$ where B has at least two elements (bounds), *i.e.* 0, 1, and $+, \cdot : B \times B \to B$ are binary operations on B, and $\bar{} : B \to B$ is a unary operation on B for which the following axioms hold for all $a, b, c \in B$:

1)	$(a + b) + c = a + (b + c)$,	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	(associativity)
2)	$a + b = b + a$,	$a \cdot b = b \cdot a$	(commutativity)
3)	$(a + b) \cdot a = a$,	$(a \cdot b) + a = a$	(absorption)
4)	$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$,	$a + (b \cdot c) = (a + b) \cdot (a + c)$	(distributivity)
5)	$a + (b \cdot \bar{b}) = a$,	$a \cdot (b + \bar{b}) = a$	

If only the first three axioms are satisfied, the structure is called a lattice. If the first four are satisfied, it is called a distributive lattice.

Show that the set of fuzzy truth values (the real interval $[0, 1]$) together with the standard fuzzy operations $\top(a, b) = \min\{a, b\}$ (conjunction), $\bot(a, b) = \max\{a, b\}$ (disjunction) and $\sim a = 1 - a$ (negation) is a distributive lattice but not a Boolean algebra.

Assignment 6 Linguistic Terms

Assume you were told that the room temperature is *around* $20°C$. How would you represent this piece of information by

a) a set and

b) a fuzzy set?

Assignment 7 Linguistic Terms

The middle point of a line segment is, at the same time, *close to* and *far from* its extreme points. How would you geometrically depict this idea through

a) sets and

b) fuzzy sets?

Fuzzy Systems
Prof. Dr. Rudolf Kruse, Christoph Doell

Assignment 8 Membership Function

Given the fuzzy set μ with the following membership function

$$\mu(x) = \begin{cases} x - 5, & \text{if } 5 \leq x \leq 6 \\ 7 - x, & \text{if } 6 \leq x \leq 7 \\ 0, & \text{otherwise.} \end{cases}$$

a) Sketch the graph of the function.

b) What are the possible semantics of this fuzzy set?