

## Assignment Sheet 10

### Assignment 35      Quantifiers

Assuming the same conditions as in Assignment 26: To describe the concept “ $x$  is a small number”, let  $x \in \mathbb{N} \cup \{0\}$  and two membership functions  $\mu_1(x)$  and  $\mu_2(x)$  be defined as follows:

$$\mu_1(x) = \begin{cases} \frac{20-x}{20}, & \text{if } x < 20 \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_2(x) = 0.95^x$$

What fuzzy truth values do you get for the proposition “There exists a small single-digit prime number in the decimal number system”? Which problem arises thereby?

Hint: Directly use the dual  $t$ -conorms  $\perp_{\max}(a, b) = \max\{a, b\}$  and  $\perp_{\text{sum}}(a, b) = a + b - a \cdot b$ .

### Assignment 36      Takagi-Sugeno Controller

Construct a Takagi-Sugeno controller with two inputs and one output that computes the following (partially defined) function (cf. Assignment 33):

$$\begin{aligned} (1, 0) &\mapsto 2, & (1, 3) &\mapsto 4, \\ (0, 2) &\mapsto 2, & (2, 2) &\mapsto 4, \\ (2, 0) &\mapsto 2. \end{aligned}$$

Determine the output of your controller for the inputs  $(1, 1)$  and  $(1.5, 1.5)$ .

### Assignment 37      Takagi-Sugeno Controller

Consider the following definition of triangular fuzzy numbers

$$\mu_{l,m,r} = \begin{cases} \frac{x-l}{m-l} & \text{if } l \leq x \leq m, \\ \frac{r-x}{r-m} & \text{if } m \leq x \leq r, \\ 0 & \text{otherwise} \end{cases}$$

whereas  $l, m, r \in \mathbb{R}$  and  $l < m < r$ . Now, let a Takagi-Sugeno controller with the rule base be given as follows

$$\begin{aligned} R_1 &: \text{if } x \text{ is } \mu_1 \text{ then } y = 2, \\ R_2 &: \text{if } x \text{ is } \mu_2 \text{ then } y = x, \\ R_3 &: \text{if } x \text{ is } \mu_3 \text{ then } y = 3 - x^2, \end{aligned}$$

whereas  $x \in X = [0, 8]$  and  $X$  is partitioned by  $\mu_1 = \mu_{0,2,4}$ ,  $\mu_2 = \mu_{2,4,6}$ ,  $\mu_3 = \mu_{4,6,8}$ .

## Fuzzy Systems

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- a) Compute the output of the controller by using the weighted sum

$$f(x) = \frac{\sum_{r=1}^3 \mu_{R_r}(x) \cdot f_{R_r}(x)}{\sum_{r=1}^3 \mu_{R_r}(x)},$$

whereas  $\mu_{R_r}(x)$  is the degree of fulfillment that the rule  $R_r$  “fires”, and  $f_{R_r}$  is the output of the rule  $R_r$ .

- b) Draw the output into a diagram.

### Assignment 38      Fuzzy Clustering

Consider the one-dimensional data set

$$1, 3, 4, 5, 8, 10, 11, 12.$$

We want to process this data set with fuzzy  $c$ -means clustering using  $c = 2$  (two clusters) and the fuzzifier  $m = 2$ . Assume that the cluster centers are initialized to 1 and 5. Execute one step of alternating optimization as it is used for fuzzy clustering, *i.e.*

- a) Compute the membership degrees of the data points for the initial cluster centers.
- b) Compute new cluster centers from the membership degrees that have been obtained before.