

Assignment Sheet 8

Assignment 28 Fuzzy Relational Equations

Let $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3, y_4\}$ be two sets and $\mu : X \rightarrow [0, 1]$ and $\nu : Y \rightarrow [0, 1]$ two fuzzy sets on X and Y , respectively, which are defined as follows:

$$\begin{aligned} \mu(x_1) &= 0.1, & \mu(x_2) &= 0.7, & \mu(x_3) &= 1.0, \\ \nu(y_1) &= 0.4, & \nu(y_2) &= 1.0, & \nu(y_3) &= 0.8, & \nu(y_4) &= 0.3. \end{aligned}$$

- a) How can you find out whether the relational equation $\mu \circ \varrho = \nu$ has a solution, *i.e.* whether there is a fuzzy relation ϱ that satisfies this equation?
- b) If the relational equation $\mu \circ \varrho = \nu$ has a solution, determine a solution. Are there other solutions than the one you found?

Assignment 29 Fuzzy Relational Equations

Let $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$ be two sets. Consider the fuzzy sets μ_1, μ_2, μ_3 on X and ν_1, ν_2, ν_3 on Y which are defined as shown in the two tables below.

	x_1	x_2	x_3			y_1	y_2
μ_1	1.0	0.6	0.2		ν_1	1.0	0.4
μ_2	0.0	0.8	1.0		ν_2	0.6	1.0
μ_3	0.9	0.1	0.0		ν_3	0.9	0.5

- a) Show that the system consisting of the two relational equations $\mu_1 \circ \varrho = \nu_1$ and $\mu_2 \circ \varrho = \nu_2$ has a solution. Find the greatest solution of this system.
- b) Is the fuzzy relation that can be computed as the union (maximum) of the two Cartesian products $\mu_1 \otimes \nu_1$ and $\mu_2 \otimes \nu_2$ also a solution of the system of relational equations considered in a)?
- c) Show that the system consisting of the three relational equations $\mu_i \circ \varrho = \nu_i$, $i = 1, 2, 3$, does not have any solution.

Fuzzy Systems

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Assignment 30 Fuzzy Control based on Relational Equations

Let $X = \{1, 2, 3\}$ and $Y = \{10, 20, 30\}$ be two sets, μ_1, μ_2 fuzzy sets on X , and ν_1, ν_2 fuzzy sets on Y , which are defined as shown in the two tables below.

	1	2	3		10	20	30
μ_1	0.0	0.5	1.0	ν_1	0.0	0.6	1.0
μ_2	1.0	0.4	0.0	ν_2	1.0	0.3	0.0

Consider a fuzzy controller with the following rule base:

if x is μ_1 then y is ν_1 ,

if x is μ_2 then y is ν_2 .

Use the Gödel relation to determine the fuzzy output of this controller for the fuzzy input $(1 : 0.1, 2 : 1, 3 : 0)$.