Fuzzy Systems

Mamdani-Assilian Controller

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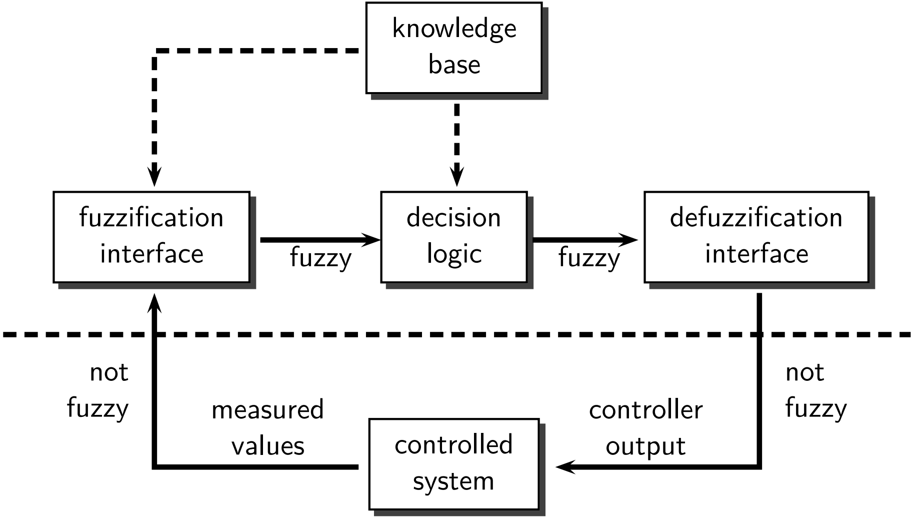
Outline

Architecture of a Fuzzy Controller



Example: Cartpole Problem (cont.)

X_1 is partitioned into 7 fuzzy sets.

Support of fuzzy sets: intervals with length $\frac{1}{4}$ of whole range X_1.

Similar fuzzy partitions for X_2 and Y.

Next step: specify rules

$$\text{if } \xi_1 \text{ is } A^{(1)} \text{ and } \ldots \text{ and } \xi_n \text{ is } A^{(n)} \text{ then } \eta \text{ is } B,$$

$A^{(1)}, \ldots, A^{(n)}$ and B represent linguistic terms corresponding to $\mu^{(1)}, \ldots, \mu^{(n)}$ and μ according to $X_1, \ldots, X_n$ and Y.

Let the rule base consist of k rules.

Example: Cartpole Problem (cont.)

					θ			
		nb	nm	ns	az	ps	pm	pb
	nb			ps	pb			
	nm				pm			
	ns	nm		ns	ps			
$\dot{\theta}$	az	nb	nm	ns	az	ps	pm	pb
	ps				ns	ps		pm
	pm				nm			
	pb				nb	ns		

19 rules for cartpole problem, *e.g.*

If θ is *approximately zero* and $\dot{\theta}$ is *negative medium*
then F is *positive medium*.

Definition of Table-based Control Function

Measurement $(x_1, \ldots, x_n) \in X_1 \times \ldots \times X_n$ is forwarded to decision logic.

Consider rule

$$\text{if } \xi_1 \text{ is } A^{(1)} \text{ and } \ldots \text{ and } \xi_n \text{ is } A^{(n)} \text{ then } \eta \text{ is } B.$$

Decision logic computes degree to $\xi_1, \ldots, \xi_n$ fulfills premise of rule.

For $1 \leq \nu \leq n$, the value $\mu^{(\nu)}(x_\nu)$ is calculated.

Combine values conjunctively by $\alpha = \min\left\{\mu^{(1)}, \ldots, \mu^{(n)}\right\}$.

For each rule R_r with $1 \leq r \leq k$, compute

$$\alpha_r = \min\left\{\mu^{(1)}_{i_{1,r}}(x_1), \ldots, \mu^{(n)}_{i_{n,r}}(x_n)\right\}.$$

Definition of Table-based Control Function II

Output of R_r = fuzzy set of output values.

Thus "cutting off" fuzzy set μ_{i_r} associated with conclusion of R_r at α_r.

So for input $(x_1, \ldots, x_n)$, R_r implies fuzzy set

$$\begin{aligned} \mu_{x_1,\ldots,x_n}^{\text{output}(R_r)} : Y &\to [0,1], \\ y &\mapsto \min\left\{\mu_{i_{1,r}}^{(1)}(x_1), \ldots, \mu_{i_{n,r}}^{(n)}(x_n), \mu_{i_r}(y)\right\}. \end{aligned}$$

If $\mu_{i_{1,r}}^{(1)}(x_1) = \ldots = \mu_{i_{n,r}}^{(n)}(x_n) = 1$, then $\mu_{x_1,\ldots,x_n}^{\text{output}(R_r)} = \mu_{i_r}$.

If for all $\nu \in \{1, \ldots, n\}$, $\mu_{i_{1,r}}^{(\nu)}(x_\nu) = 0$, then $\mu_{x_1,\ldots,x_n}^{\text{output}(R_r)} = 0$.

Combination of Rules

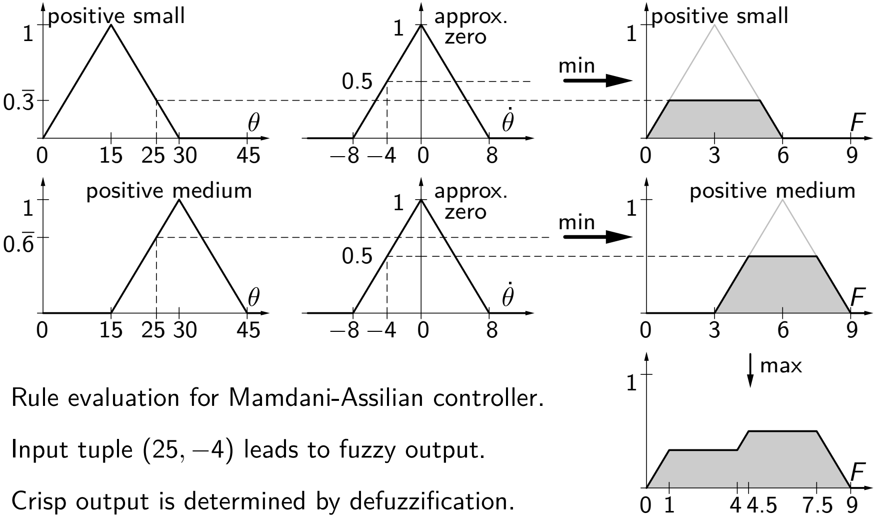
The decision logic combines the fuzzy sets from all rules.

The **maximum** leads to the output fuzzy set

$$\begin{aligned}\mu^{\text{output}}_{x_1,\ldots,x_n} : Y &\to [0,1],\\ y &\mapsto \max_{1\le r\le k}\left\{\min\left\{\mu^{(1)}_{i_{1,r}}(x_1),\ldots,\mu^{(n)}_{i_{n,r}}(x_n),\mu_{i_r}(y)\right\}\right\}.\end{aligned}$$

Then $\mu^{\text{output}}_{x_1,\ldots,x_n}$ is passed to defuzzification interface.

Rule Evaluation



Rule evaluation for Mamdani-Assilian controller.

Input tuple (25, −4) leads to fuzzy output.

Crisp output is determined by defuzzification.

Defuzzification

So far: mapping between each $(n_1, \ldots, n_n)$ and $\mu^{\text{output}}_{x_1,\ldots,x_n}$.

Output = description of output value as fuzzy set.

Defuzzification interface derives crisp value from $\mu^{\text{output}}_{x_1,\ldots,x_n}$.

This step is called **defuzzification**.

Most common methods:

- max criterion,
- mean of maxima,
- center of gravity.

The Max Criterion Method

Choose an arbitrary $y \in Y$ for which $\mu_{x_1,\ldots,x_n}^{\text{output}}$ reaches the maximum membership value.

Advantages:

- Applicable for arbitrary fuzzy sets.
- Applicable for arbitrary domain Y (even for $Y \neq \mathbb{R}$).

Disadvantages:

- Rather class of defuzzification strategies than single method.
- Which value of maximum membership?
- Random values and thus non-deterministic controller.
- Leads to discontinuous control actions.

The Mean of Maxima (MOM) Method

Preconditions:

(i) Y is interval

(ii) $Y_{\text{Max}} = \{y \in Y \mid \forall y' \in Y : \mu^{\text{output}}_{x_1,\ldots,x_n}(y') \le \mu^{\text{output}}_{x_1,\ldots,x_n}(y)\}$ is non-empty and measurable

(iii) Y_{Max} is set of all $y \in Y$ such that $\mu^{\text{output}}_{x_1,\ldots,x_n}$ is maximal

Crisp output value = mean value of Y_{Max}.

if Y_{Max} is finite:

$$\eta = \frac{1}{|Y_{\text{Max}}|} \sum_{y_i \in Y_{\text{Max}}} y_i$$

if Y_{Max} is infinite:

$$\eta = \frac{\int_{y \in Y_{\text{Max}}} y \, dy}{\int_{y \in Y_{\text{Max}}} dy}$$

MOM can lead to discontinuous control actions.

Center of Gravity (COG) Method

Same preconditions as MOM method.

$\eta =$ center of gravity/area of $\mu^{\text{output}}_{x_1,\dots,x_n}$

If Y is finite, then

$$\eta = \frac{\sum_{y_i \in Y} y_i \cdot \mu^{\text{output}}_{x_1,\dots,x_n}(y_i)}{\sum_{y_i \in Y} \mu^{\text{output}}_{x_1,\dots,x_n}(y_i)}.$$

If Y is infinite, then

$$\eta = \frac{\int_{y \in Y} y \cdot \mu^{\text{output}}_{x_1,\dots,x_n}(y)\, dy}{\int_{y \in Y} \mu^{\text{output}}_{x_1,\dots,x_n}(y)\, dy}.$$

Center of Gravity (COG) Method

Advantages:

- Nearly always smooth behavior,
- If certain rule dominates once, not necessarily dominating again.

Disadvantage:

- No semantic justification,
- Long computation,
- Counterintuitive results possible.

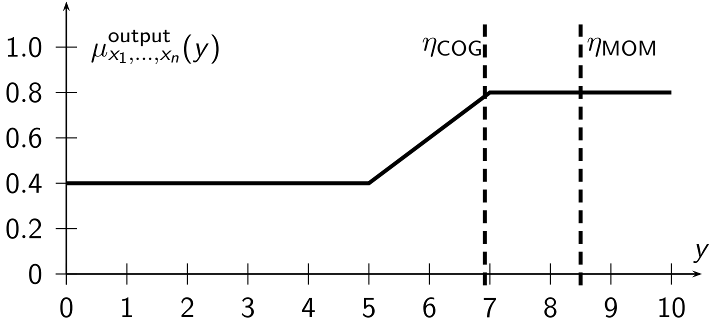
Also called *center of area (COA) method*:

take value that splits $\mu_{x_1,\ldots,x_n}^{\text{output}}$ into 2 equal parts.

Example

Task: compute η_{COG} and η_{MOM} of fuzzy set shown below.

Based on finite set $Y = 0, 1, \dots, 10$ and infinite set $Y = [0, 10]$.



Example for COG

Continuous and Discrete Output Space

$$\begin{aligned}
\eta_{\text{COG}} &= \frac{\int_0^{10} y \cdot \mu_{x_1,\ldots,x_n}^{\text{output}}(y)\, dy}{\int_0^{10} \mu_{x_1,\ldots,x_n}^{\text{output}}(y)\, dy} \\
&= \frac{\int_0^5 0.4y\, dy + \int_5^7 (0.2y - 0.6)y\, dy + \int_7^{10} 0.8y\, dy}{5 \cdot 0.4 + 2 \cdot \frac{0.8+0.4}{2} + 3 \cdot 0.8} \\
&\approx \frac{38.7333}{5.6} \approx 6.917
\end{aligned}$$

$$\begin{aligned}
\eta_{\text{COG}} &= \frac{0.4 \cdot (0+1+2+3+4+5) + 0.6 \cdot 6 + 0.8 \cdot (7+8+9+10)}{0.4 \cdot 6 + 0.6 \cdot 1 + 0.8 \cdot 4} \\
&= \frac{36.8}{6.2} \approx 5.935
\end{aligned}$$

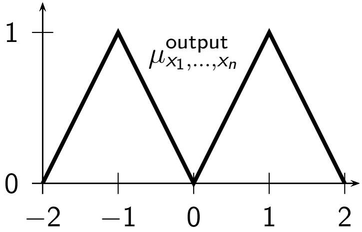
Example for MOM

Continuous and Discrete Output Space

$$\begin{aligned}\eta_{\text{MOM}} &= \frac{\int_7^{10} y\,dy}{\int_7^{10}\,dy}\\ &= \frac{50 - 24.5}{10 - 7} = \frac{25.5}{3}\\ &= 8.5\end{aligned}$$

$$\begin{aligned}\eta_{\text{MOM}} &= \frac{7 + 8 + 9 + 10}{4}\\ &= \frac{34}{4}\\ &= 8.5\end{aligned}$$

Problem Case for MOM and COG



What would be the output of MOM or COG?

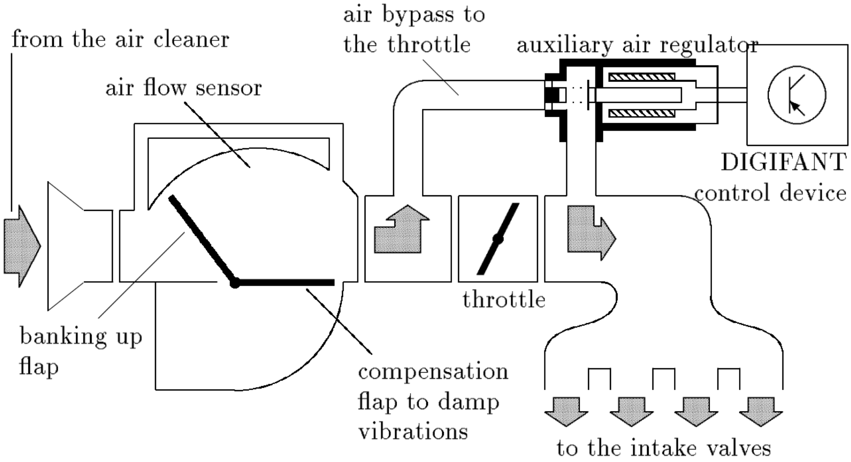
Is this desirable or not?

Outline

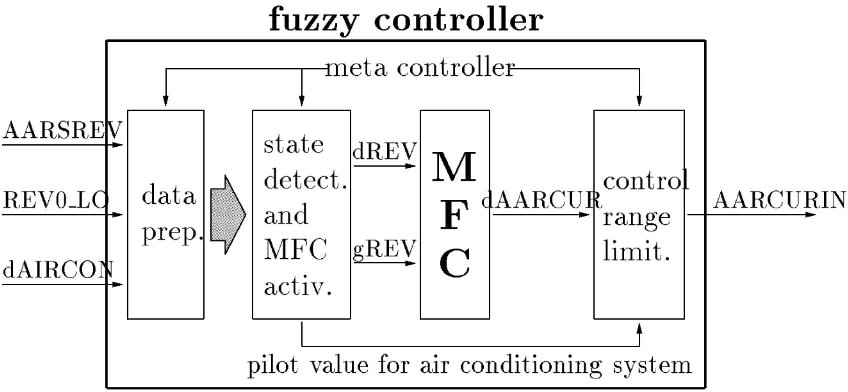
1. Motivation
2. **Example: Engine Idle Speed Control**
3. Example: Automatic Gear Box

Example: Engine Idle Speed Control

VW 2000cc 116hp Motor (Golf GTI)

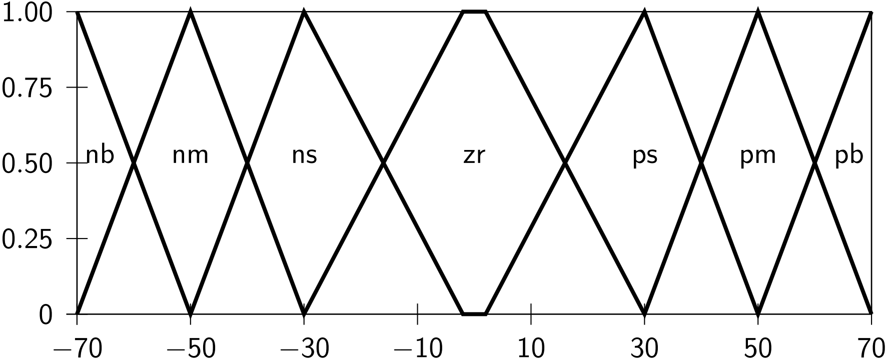


Structure of the Fuzzy Controller



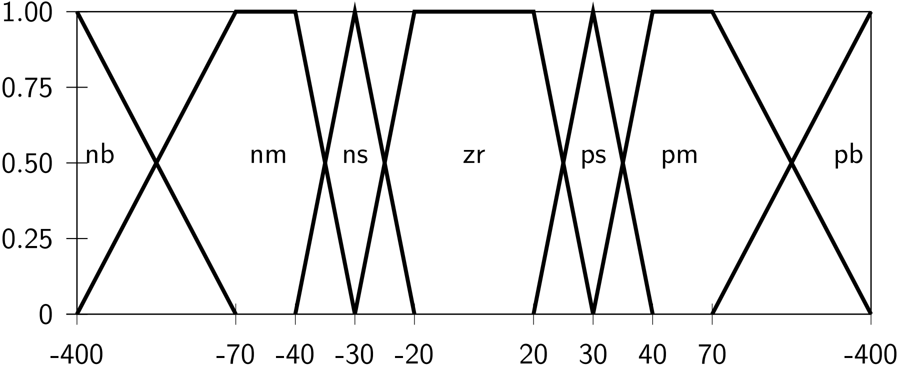
Deviation of the Number of Revolutions

dREV



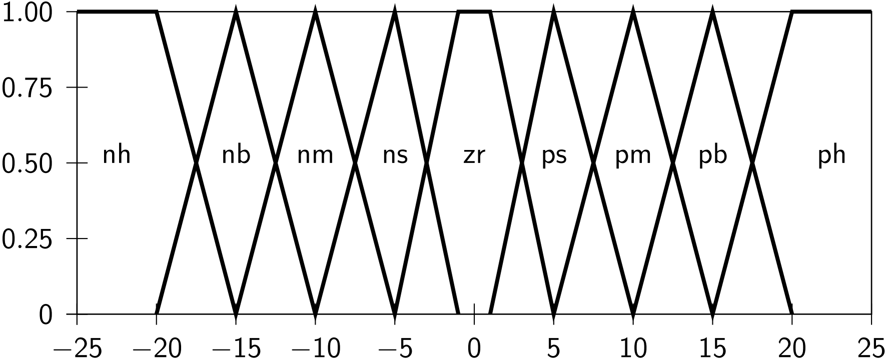
Gradient of the Number of Revolutions

gREV



Change of Current for Auxiliary Air Regulator

dAARCUR

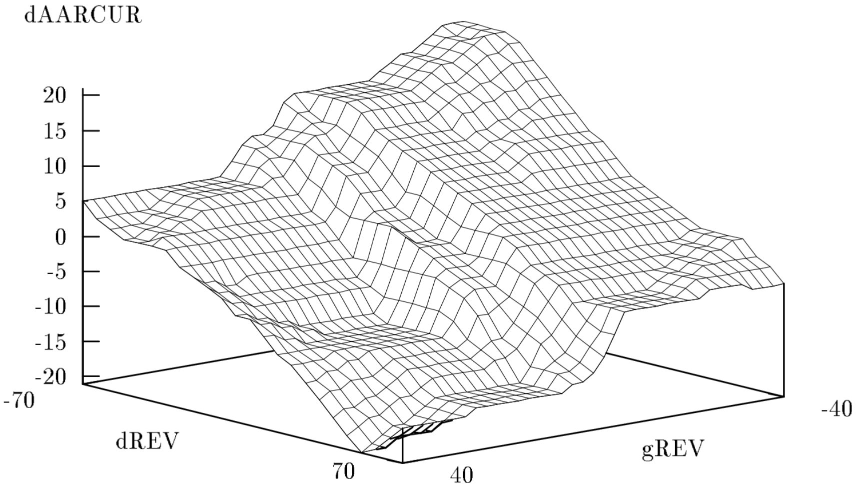


Rule Base

If the deviation from the desired number of revolutions is negative small **and** the gradient is negative medium,
then the change of the current for the auxiliary air regulation should be positive medium.

		gREV						
		nb	nm	ns	az	ps	pm	pb
dREV	nb	ph	pb	pb	pm	pm	ps	ps
	nm	ph	pb	pm	pm	ps	ps	az
	ns	pb	pm	ps	ps	az	az	az
	az	ps	ps	az	az	az	nm	ns
	ps	az	az	az	ns	ns	nm	nb
	pm	az	ns	ns	ns	nb	nb	nh
	pb	ns	ns	nm	nb	nb	nb	nh

Performance Characteristics



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2. Example: Engine Idle Speed Control
3. **Example: Automatic Gear Box**

Example: Automatic Gear Box I

VW gear box with 2 modes (eco, sport) in series line until 1994.

Research issue since 1991: individual adaption of set points and no additional sensors.

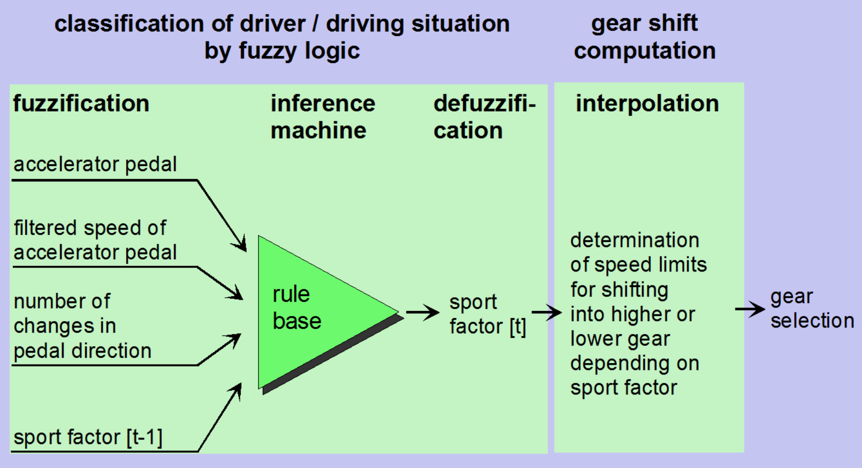
Idea: car “watches” driver and classifies him/her into calm, normal, sportive (assign sport factor $[0, 1]$), or nervous (calm down driver).

Test car: different drivers, classification by expert (passenger).

Simultaneous measurement of 14 attributes, *e.g.* , speed, position of accelerator pedal, speed of accelerator pedal, kick down, steering wheel angle.

Example: Automatic Gear Box II

Continuously Adapting Gear Shift Schedule in VW New Beetle



classification of driver / driving situation by fuzzy logic

gear shift computation

fuzzification

inference machine

defuzzification

interpolation

accelerator pedal

filtered speed of accelerator pedal

number of changes in pedal direction

sport factor [t-1]

rule base

sport factor [t]

determination of speed limits for shifting into higher or lower gear depending on sport factor

gear selection

Example: Automatic Gear Box III

Technical Details

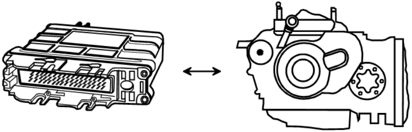
Optimized program on Digimat:

24 byte RAM

702 byte ROM

Runtime: 80 ms

12 times per second new sport factor is assigned.



Research topics:

When fuzzy control?

How to find fuzzy rules?