Fuzzy Systems
Mamdani-Assilian Controller

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Outline

1. Motivation
   - Architecture of a Fuzzy Controller
   - Cartpole Problem
   - Table-based Control Function
   - Combination of Rules
   - Defuzzification

2. Example: Engine Idle Speed Control

3. Example: Automatic Gear Box
Architecture of a Fuzzy Controller

- fuzzification interface
- decision logic
- defuzzification interface
- measured values
- controlled system
- controller output
- knowledge base

Not fuzzy
Fuzzy
Not fuzzy
Example: Cartpole Problem (cont.)

$X_1$ is partitioned into 7 fuzzy sets.

Support of fuzzy sets: intervals with length $\frac{1}{4}$ of whole range $X_1$.

Similar fuzzy partitions for $X_2$ and $Y$.

Next step: specify rules

if $\xi_1$ is $A^{(1)}$ and $\ldots$ and $\xi_n$ is $A^{(n)}$ then $\eta$ is $B$,

$A^{(1)}, \ldots, A^{(n)}$ and $B$ represent linguistic terms corresponding to $\mu^{(1)}, \ldots, \mu^{(n)}$ and $\mu$ according to $X_1, \ldots, X_n$ and $Y$.

Let the rule base consist of $k$ rules.
19 rules for cartpole problem, e.g.

If $\theta$ is *approximately zero* and $\dot{\theta}$ is *negative medium* then $F$ is *positive medium*.
Definition of Table-based Control Function

Measurement \((x_1, \ldots, x_n) \in X_1 \times \ldots \times X_n\) is forwarded to decision logic.

Consider rule

\[
\text{if } \xi_1 \text{ is } A^{(1)} \text{ and } \ldots \text{ and } \xi_n \text{ is } A^{(n)} \text{ then } \eta \text{ is } B.
\]

Decision logic computes degree to \(\xi_1, \ldots, \xi_n\) fulfills premise of rule.

For \(1 \leq \nu \leq n\), the value \(\mu^{(\nu)}(x_\nu)\) is calculated.

Combine values conjunctively by \(\alpha = \min \{\mu^{(1)}, \ldots, \mu^{(n)}\}\).

For each rule \(R_r\) with \(1 \leq r \leq k\), compute

\[
\alpha_r = \min \left\{\mu_{i_1,r}^{(1)}(x_1), \ldots, \mu_{i_n,r}^{(n)}(x_n)\right\}.
\]
Definition of Table-based Control Function II

Output of $R_r = \text{fuzzy set of output values.}$

Thus “cutting off” fuzzy set $\mu_{i_r}$ associated with conclusion of $R_r$ at $\alpha_r$.

So for input $(x_1, \ldots, x_n)$, $R_r$ implies fuzzy set

$$
\mu_{x_1, \ldots, x_n}^{\text{output}}(R_r) : Y \rightarrow [0, 1],
$$

$$
y \mapsto \min \left\{ \mu_{i_1,r}^{(1)}(x_1), \ldots, \mu_{i_n,r}^{(n)}(x_n), \mu_{i_r}(y) \right\}.
$$

If $\mu_{i_1,r}^{(1)}(x_1) = \ldots = \mu_{i_n,r}^{(n)}(x_n) = 1$, then $\mu_{x_1,\ldots,x_n}^{\text{output}}(R_r) = \mu_{i_r}$.

If for all $\nu \in \{1, \ldots, n\}$, $\mu_{i_1,r}^{(\nu)}(x_{\nu}) = 0$, then $\mu_{x_1,\ldots,x_n}^{\text{output}}(R_r) = 0$. 
Combination of Rules

The decision logic combines the fuzzy sets from all rules.

The **maximum** leads to the output fuzzy set

\[
\mu^{\text{output}}_{x_1, \ldots, x_n} : Y \rightarrow [0, 1],
\]

\[
y \mapsto \max_{1 \leq r \leq k} \left\{ \min \left\{ \mu^{(1)}_{i_1, r}(x_1), \ldots, \mu^{(n)}_{i_n, r}(x_n), \mu_{i_r}(y) \right\} \right\}.
\]

Then \( \mu^{\text{output}}_{x_1, \ldots, x_n} \) is passed to defuzzification interface.
Rule Evaluation

Rule evaluation for Mamdani-Assilian controller.

Input tuple (25, −4) leads to fuzzy output.

Crisp output is determined by defuzzification.
Defuzzification

So far: mapping between each \((n_1, \ldots, n_n)\) and \(\mu_{x_1,\ldots,x_n}\).

Output = description of output value as fuzzy set.

Defuzzification interface derives crisp value from \(\mu_{x_1,\ldots,x_n}\).

This step is called **defuzzification**.

Most common methods:

- max criterion,
- mean of maxima,
- center of gravity.
The Max Criterion Method

Choose an arbitrary \( y \in Y \) for which \( \mu_{\text{output} \, x_{1},...,x_{n}} \) reaches the maximum membership value.

Advantages:

- Applicable for arbitrary fuzzy sets.
- Applicable for arbitrary domain \( Y \) (even for \( Y \neq \mathbb{IR} \)).

Disadvantages:

- Rather class of defuzzification strategies than single method.
- Which value of maximum membership?
- Random values and thus non-deterministic controller.
- Leads to discontinuous control actions.
The Mean of Maxima (MOM) Method

Preconditions:

(i) $Y$ is interval

(ii) $Y_{\text{Max}} = \{ y \in Y \mid \forall y' \in Y : \mu_{x_1, \ldots, x_n}^{\text{output}}(y') \leq \mu_{x_1, \ldots, x_n}^{\text{output}}(y) \}$ is non-empty and measurable

(iii) $Y_{\text{Max}}$ is set of all $y \in Y$ such that $\mu_{x_1, \ldots, x_n}^{\text{output}}$ is maximal

Crisp output value = mean value of $Y_{\text{Max}}$.

if $Y_{\text{Max}}$ is finite:

$$\eta = \frac{1}{|Y_{\text{Max}}|} \sum_{y_i \in Y_{\text{Max}}} y_i$$

if $Y_{\text{Max}}$ is infinite:

$$\eta = \frac{\int_{y \in Y_{\text{Max}}} y \, dy}{\int_{y \in Y_{\text{Max}}} dy}$$

MOM can lead to discontinuous control actions.
Center of Gravity (COG) Method

Same preconditions as MOM method.

\[ \eta = \text{center of gravity/area of } \mu_{x_1, \ldots, x_n}^{\text{output}} \]

If \( Y \) is finite, then

\[ \eta = \frac{\sum_{y_i \in Y} y_i \cdot \mu_{x_1, \ldots, x_n}^{\text{output}}(y_i)}{\sum_{y_i \in Y} \mu_{x_1, \ldots, x_n}^{\text{output}}(y_i)} \]

If \( Y \) is infinite, then

\[ \eta = \frac{\int_{y \in Y} y \cdot \mu_{x_1, \ldots, x_n}^{\text{output}}(y) \, dy}{\int_{y \in Y} \mu_{x_1, \ldots, x_n}^{\text{output}}(y) \, dy} \]
Center of Gravity (COG) Method

Advantages:

- Nearly always smooth behavior,
- If certain rule dominates once, not necessarily dominating again.

Disadvantage:

- No semantic justification,
- Long computation,
- Counterintuitive results possible.

Also called center of area (COA) method:

take value that splits $\mu_{\text{output}, x_1, \ldots, x_n}$ into 2 equal parts.
Example

Task: compute $\eta_{\text{COG}}$ and $\eta_{\text{MOM}}$ of fuzzy set shown below.
Based on finite set $Y = 0, 1, \ldots, 10$ and infinite set $Y = [0, 10]$. 
Example for COG
Continuous and Discrete Output Space

\[ \eta_{\text{COG}} = \frac{\int_0^{10} y \cdot \mu_{x_1,\ldots,x_n}(y) \, dy}{\int_0^{10} \mu_{x_1,\ldots,x_n}(y) \, dy} \]

\[ = \frac{\int_0^5 0.4y \, dy + \int_5^7 (0.2y - 0.6)y \, dy + \int_7^{10} 0.8y \, dy}{5 \cdot 0.4 + 2 \cdot \frac{0.8 + 0.4}{2} + 3 \cdot 0.8} \]

\[ \approx \frac{38.7333}{5.6} \approx 6.917 \]

\[ \eta_{\text{COG}} = \frac{0.4 \cdot (0 + 1 + 2 + 3 + 4 + 5) + 0.6 \cdot 6 + 0.8 \cdot (7 + 8 + 9 + 10)}{0.4 \cdot 6 + 0.6 \cdot 1 + 0.8 \cdot 4} \]

\[ = \frac{36.8}{6.2} \approx 5.935 \]
Example for MOM
Continuous and Discrete Output Space

\[ \eta_{\text{MOM}} = \frac{\int_{7}^{10} y \, dy}{\int_{7}^{10} dy} \]

\[ = \frac{50 - 24.5}{10 - 7} = \frac{25.5}{3} \]

\[ = 8.5 \]

\[ \eta_{\text{MOM}} = \frac{7 + 8 + 9 + 10}{4} \]

\[ = \frac{34}{4} \]

\[ = 8.5 \]
Problem Case for MOM and COG

What would be the output of MOM or COG?
Is this desirable or not?
Outline

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2. Example: Engine Idle Speed Control

3. Example: Automatic Gear Box
Example: Engine Idle Speed Control
VW 2000cc 116hp Motor (Golf GTI)

from the air cleaner

air flow sensor

air bypass to the throttle

auxiliary air regulator

DIGIFANT control device

to the intake valves

compensation flap to damp vibrations

banking up flap
Structure of the Fuzzy Controller

fuzzy controller

meta controller

AARSREV
REV0_LO
daAIRCON

data prep.

state detect. and MFC activ.

dREV
gREV

dAARCUR

control range limit.

gREVC

pilot value for air conditioning system

AARCURIN
Deviation of the Number of Revolutions

dREV

(nb, nm, ns, zr, ps, pm, pb)
Gradient of the Number of Revolutions
\( g_{\text{REV}} \)
Change of Current for Auxiliary Air Regulator
dAARCUR

R. Kruse, C. Doell

FS – Mamdani-Assilian Controller

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Rule Base

If the deviation from the desired number of revolutions is negative small and the gradient is negative medium, then the change of the current for the auxiliary air regulation should be positive medium.

gREV

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Performance Characteristics

dAARCUR

dREV
70
40
gREV
-70
-40
-20
-15
-10
-5
0
5
10
15
20
Outline

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3. Example: Automatic Gear Box
Example: Automatic Gear Box I

VW gear box with 2 modes (eco, sport) in series line until 1994. Research issue since 1991: individual adaption of set points and no additional sensors.

Idea: car “watches” driver and classifies him/her into calm, normal, sportive (assign sport factor $[0, 1]$), or nervous (calm down driver).

Test car: different drivers, classification by expert (passenger). Simultaneous measurement of 14 attributes, e.g., speed, position of accelerator pedal, speed of accelerator pedal, kick down, steering wheel angle.
Example: Automatic Gear Box II
Continuously Adapting Gear Shift Schedule in VW New Beetle

- **fuzzification**
  - accelerator pedal
  - filtered speed of accelerator pedal
  - number of changes in pedal direction
  - sport factor [t-1]

- **inference machine**
  - rule base

- **defuzzification**
  - sport factor [t]

- **interpolation**
  - determination of speed limits for shifting into higher or lower gear depending on sport factor

- **gear selection**

**Classification of driver / driving situation by fuzzy logic**
Example: Automatic Gear Box III
Technical Details

Optimized program on Digimat:
24 byte RAM
702 byte ROM

Runtime: 80 ms
12 times per second new sport factor is assigned.

Research topics:
When fuzzy control?
How to find fuzzy rules?