Fuzzy Systems

Introduction

Prof. Dr. Rudolf Kruse **Christoph Doell**

{kruse,doell}@iws.cs.uni-magdeburg.de
Otto-von-Guericke University of Magdeburg
Faculty of Computer Science
Department of Knowledge Processing and Language Engineering

Content of the lecture

Introduction

Fuzzy Set Theory

Fuzzy Set Operators

Fuzzy Arithmetic

Fuzzy Relations

Fuzzy Rule Bases

Mamdani-Assilian Controller

Takagi-Sugeno and Similarity-based Controllers

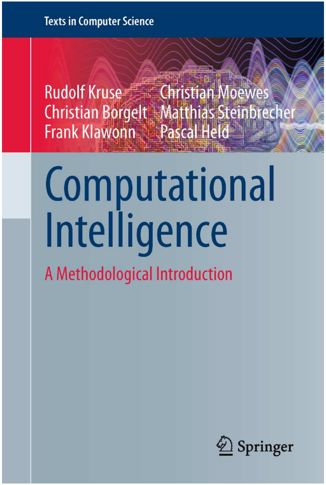
Fuzzy Clustering (two lectures)

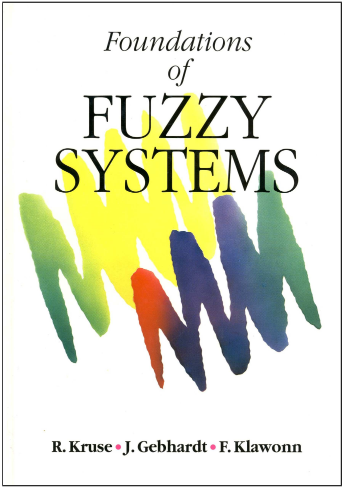
Neuro-Fuzzy Systems

Evolutionary Fuzzy Systems

Possibility Theory

Books about the course





http://www.computational-intelligence.eu/

What are we going to talk about?!

Research on fuzzy systems wants to establish

- theoretical and methodological bases for computational intelligence,
- tools and techniques for design of intelligent systems.

Fuzzy systems focus on applications

- where imprecision and uncertainty play an important role.

Fuzzy set theory and fuzzy logic

- supplies us with the basic mathematical foundation.

Outline

1. Motivation

Imprecision

Uncertainty

2. Fuzzy Sets

Motivation

Every day humans use imprecise linguistic terms
e.g. big, *fast*, *about 12 o'clock*, *old*, etc.

All complex human actions are decisions based on such concepts:

- driving and parking a car,
- financial/business decisions,
- law and justice,
- giving a lecture,
- listening to the professor/tutor.

So, these terms and the way they are processed play a crucial role.

Computers need a mathematical model to express and process such complex semantics.

Concepts in classical mathematics are inadequate for such models.

Lotfi Asker Zadeh (1965)

Classes of objects in the real world do not have precisely defined criteria of membership.

Such imprecisely defined "classes" play important role in human thinking,

particularly in domains of pattern recognition, communication of information, and abstraction.

Zadeh in 2004 (born 1921)

Imprecision

Any notion is said to be imprecise when its *meaning* is not fixed by sharp boundaries.

Can be applied fully/to certain degree/not at all.

Gradualness (“membership gradience”) also called *fuzziness*.

Proposition is imprecise if it contains gradual predicates.

Such propositions may be neither true nor false, but in-between.

They are true *to a certain degree*, *i.e.* partial truth.

Forms of such degrees can be found in natural language, *e.g. very*, *rather*, *almost not*, etc.

Example I – The Sorites Paradox

If a sand dune is small, adding one grain of sand to it leaves it small.
A sand dune with a single grain is small.

Hence all sand dunes are small.

Paradox comes from all-or-nothing treatment of *small*.

Degree of truth of "heap of sand is small" decreases by adding one grain after another.

Certain number of words refer to continuous numerical scales.

Example I – The Sorites Paradox

How many grains of sand has a sand dune at least?

Statement $A(n)$: “n grains of sand are a sand dune.”

Let $d_n = T(A(n))$ denote “degree of acceptance” for $A(n)$.

Then

$$0 = d_0 \leq d_1 \leq \ldots \leq d_n \leq \ldots \leq 1$$

can be seen as truth values of a **many valued logic**.

Why is there imprecision in all languages?

Why is there imprecision?

Any language is discrete and real world is continuous!

Gap between discrete representation and continuous perception,
i.e. prevalence of ambiguity in languages.

Consider the word *young*, applied to humans.

The more fine-grained the scale of age,
e.g. going from years to months, weeks, days, etc.,
the more difficult is it to fix threshold
below which *young* fully applies,
above which *young* does not at all.

Conflict between linguistic and numerical representation:
finite term set $\{\text{young}, \text{mature}, \text{old}\}$,
real-valued interval $[0, 140]$ years for humans.

Imprecision

Is there a membership threshold for imprecisely defined classes?

Consider the notion *bald*:
A man without hair on his head is bald,
a hairy man is not bald.

Usually, *bald* is only partly applicable.

Where to set *baldness/non baldness* threshold?

Fuzzy set theory does not assume any threshold!

This has consequences for the logic behind fuzzy set theory.

To be discussed in this course later.

Uncertainty

Imprecision refers to contents of a piece of information.

Uncertainty is different from imprecision.

Uncertainty describes the ability of an agent to claim whether a proposition holds or not.

Several kinds of uncertainty:

- Uncertainty modeled by propositional logic or possibility theory.
- Uncertainty modeled by probability.

Uncertainty modeled by Probability

Uncertainty also comes from conflicting but precisely observed pieces of information.

Usually in statistics: consider random experiment run several times and not producing same outcomes.

Uncertainty due to lack of information.

Distinction between Imprecision and Uncertainty

Imprecision:

e.g. "Today the weather is fine."

Imprecisely defined concepts

neglect of details

computing with words

Uncertainty:

e.g. "How will the exchange rate of the dollar be tomorrow?"

probability, possibility

Examples of Imprecision and Uncertainty

Uncertainty differs from imprecision. It can result from it.

"This car is rather old." (imprecision)
Lack of ability to measure or to evaluate numerical features.

"This car was probably made in Germany." (uncertainty)
Uncertainty about well-defined proposition *made in Germany*, perhaps based on statistics (random experiment).

"The car I chose randomly is perhaps very big." (uncertainty and imprecision)
Lack of precise definition of notion *big*.
Modifier *very* indicates rough degree of "bigness".

Lotfi A. Zadeh's Principle of Incompatibility

> *"Stated informally, the essence of this principle is that as the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics."*

Fuzzy sets/fuzzy logic are used as mechanism for abstraction of unnecessary or too complex details.

Applications of Fuzzy Systems

Control Engineering

Approximate Reasoning

Data Analysis

Image Analysis

Advantages:

Use of imprecise or uncertain information

Use of expert knowledge

Robust nonlinear control

Time to market

Marketing aspects

Washing Machines Use Fuzzy Logic

Source: `http://www.siemens-home.com/`

Outline

1. Motivation

2. **Fuzzy Sets**

   Membership Functions

   Fuzzy Numbers

   Linguistic Variables and Linguistic Values

   Semantics

Membership Functions I

Lotfi A. Zadeh (1965)
"A fuzzy set is a class with a continuum of membership grades."

Fuzzy set M is characterized by membership function μ_M.

μ_M associates real number in $[0, 1]$ with each element $x \in X$.

Value of μ_M at x represents *grade of membership* of x in M.

Fuzzy set M is thus defined as mapping

$$\mu_M : X \mapsto [0, 1].$$

So, μ_M generalizes traditional characteristic function

$$\chi_M : X \mapsto \{0, 1\}.$$

Membership Functions II

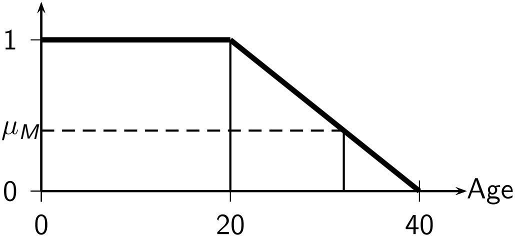
$\mu_M(u) = 1$ reflects full membership in M.

$\mu_M(u) = 0$ expresses absolute non-membership in M.

Sets can be viewed as special case of fuzzy sets where only full membership and absolute non-membership are allowed.

Such sets are called *crisp sets* or Boolean sets.

Membership degrees $0 < \mu_M < 1$ represent *partial membership*.



Representing *young* in “a young person”

Membership Functions III

Membership function attached to given word (such as *young*) depends on context:
Young retired person is certainly older than young student.
Even idea of young student depends on the user.

Membership degrees are fixed only *by convention*:
Unit interval as range of membership grades is arbitrary.
Natural for modeling membership grades of fuzzy sets of real numbers.

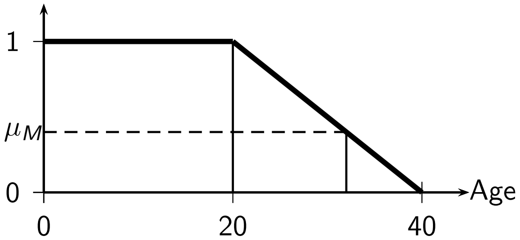
OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG INF FAKULTÄT FÜR INFORMATIK

Membership Functions IV

Consider again representation for predicate *young*

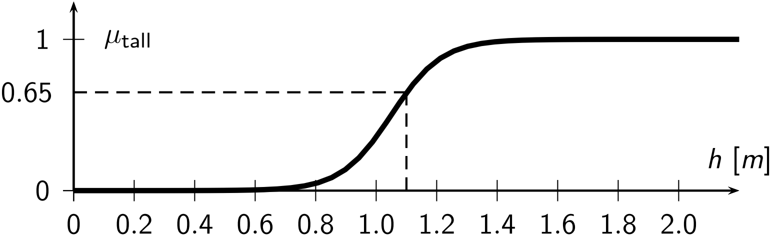
There is no precise threshold between
prototypes of *young* and
prototypes of *not young*.

Fuzzy sets offer natural interface between linguistic and numerical representations.



Representing *young* in “a young person”

Example V – Body Height of 4 Year Old Boys



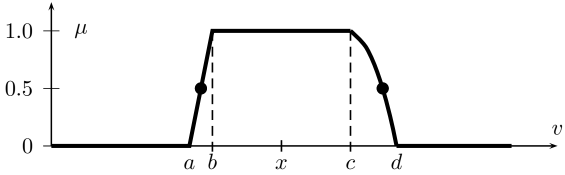
Membership function for μ_{tall}

1.5 m is for sure tall, 0.7 m is for sure small, *but in-between?!*

Imprecise predicate *tall* modeled as sigmoid function,
e.g. height of 1.1 m has membership degree of 0.65.

So, height of 1.1 m satisfies predicate *tall* with 0.65.

Example VI – Velocity of Rotating Hard Disk



Fuzzy set μ characterizing velocity of rotating hard disk.

Let x be velocity v of rotating hard disk in revolutions per minute.

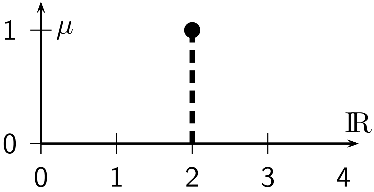
If no observations about x available, use expert's knowledge:
"It's *impossible* that v drops under a or exceeds d.
"It's highly certain that any value between $[b, c]$ can occur."

Additionally, values of v with membership degree of 0.5 are provided.

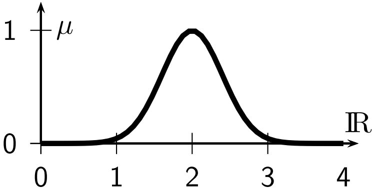
Interval $[a, d]$ is called *support* of the fuzzy set.

Interval $[b, c]$ is denoted as *core* of the fuzzy set.

Examples for Fuzzy Numbers



exactly two



around two

Exact numerical value has membership degree of 1.

Left: monotonically increasing, right: monotonically decreasing, *i.e.* unimodal function.

Terms like *around* modeled using triangular or Gaussian function.

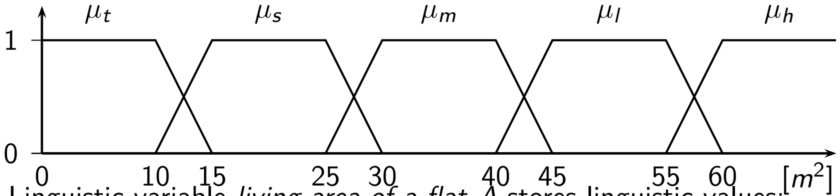
Linguistic Variables and Linguistic Values

Linguistic variables represent attributes in fuzzy systems.

They are **partitioned** into **linguistic values** (not numerical!).

Partition is usually chosen subjectively (based on human intuition).

All linguistic values have a meaning, not a precise numerical value.



Linguistic variable *living area of a flat A* stores linguistic values: *e.g. tiny*, *small*, *medium*, *large*, *huge*

Every $x \in A$ has $\mu(x) \in [0, 1]$ to each value, *e.g.* let $a = 42.5m^2$. So, $\mu_t(a) = \mu_s(a) = \mu_h(a) = 0$, $\mu_m(a) = \mu_l(a) = 0.5$.

Semantics of Fuzzy Sets

What membership grades may mean?

Fuzzy sets are relevant in three types of information-driven tasks:

classification and data analysis,

decision-making problems,

approximate reasoning.

These three tasks exploit three semantics of membership grades:

similarity

preference

uncertainty

Degree of Similarity

Oldest interpretation of membership grades.

$\mu(u)$ is degree of proximity of u from prototype elements of μ.

Goes back to interests of fuzzy set concept in pattern classification.

Still used today for cluster analysis, regression, etc.
Here, proximity between pieces of information is modeled.

Also, in fuzzy control: similarity degrees are measured between current situation and prototypical ones.

Degree of Preference

μ represents both
set of more or less preferred objects and
values of a decision variable X.

$\mu(u)$ represents both
intensity of preference in favor of object u and
feasibility of selecting u as value of X.

Fuzzy sets then represent criteria or flexible constraints.

This has been used in
fuzzy optimization (especially fuzzy linear programming) and
decision analysis.

Typical applications: engineering design and scheduling problems.

Degree of Possibility

This interpretation was implicitly proposed by Zadeh when he introduced possibility theory and
developed his theory of approximate reasoning.

$\mu(u)$ can be viewed as
degree of possibility that parameter X has value u
given the only information "X is μ".

Then support values are mutually exclusive and
membership degrees rank these values by their possibility.

This view has been used in expert systems and artificial intelligence.

Semantics of Fuzzy Sets – Examples

Classifying cars of known dimensions into *big*, *regular*, *small*:
Computation of membership degree of each car to category *big* by choosing prototype of big car and
measuring distance between car and prototype.

Buying a big car:
Membership grade of our dream to class of big cars
$=$ degree of satisfaction according to criterion "size".

Somebody says that (s)he just saw a big car (so *what is known*):
Membership grade of car to class of big cars
$=$ degree of possibility that this car is same one observed.
High membership degree: confidence that car is known might be low.
Low membership degree: impossible candidate can be rejected.