Fuzzy Systems
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Assignment Sheet 9

Assignment 31 System of Relational Equations

Saint Nicholas has somewhat read the manual of his Robopet in the meantime. To tryout what has been learned already, he wants to teach the pet how to greet the reindeers and him. Of course, the pet shall be responsive much stronger to Nicholas than to the reindeers. The recognition shall be performed with embedded color sensors. They measure how similar incoming light is to the respective elementary colors.

Saint Nicholas uses the basic set $C = \{r, g, b\}$ (red, green, blue) and $E = \{a, p, j\}$ (attention, protective instinct, joy). Furthermore, he defines fuzzy sets $\mu_1 : C \to [0,1]$, $\mu_2 : C \to [0,1]$ and $\nu_1 : E \to [0,1]$, $\nu_2 : E \to [0,1]$ as shown in the two tables below.

	r	g	b
μ_1	1.0	0.1	0.1
μ_2	0.5	0.2	0.3

	a	p	j
ν_1	0.2	0.6	1.0
ν_2	0.2	1.0	0.2

Finally, he creates a controller based on Gödel's implication with the following rules

if c **is** μ_1 **then** e **is** ν_1**,**
if c **is** μ_2 **then** e **is** ν_2**,**

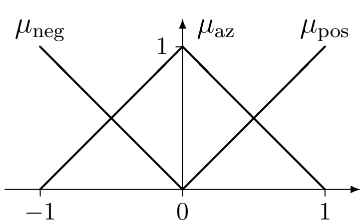
where $c \in C$ and $e \in E$.

a) Why might Nicholas not be fully satisfied with the result? How can he avoid this problem?

b) Use the Gödel relation to compute the fuzzy output value of the controller when Robopet sees the waste collection vehicle with $\mu_1(r) = \mu_1(g) = \mu_1(b) = 0.6$.

Assignment 32 Mamdani-Assilian Controller

Consider a Mamdani-Assilian controller with two inputs $\xi_1 \in X_1 = [-1, 1]$ and $\xi_2 \in X_2 = [-1, 1]$ and one output $\eta \in Y = [-1, 1]$. The utilized fuzzy partitions shall be the same for all three domains. They are shown below on the left ("az" means "approximately zero"). The rule base of the controller is shown on the right in tabular form.



ξ_2 \ ξ_1	neg	az	pos
neg	neg		az
az		az	
pos	az		pos

a) Determine the fuzzy output of this controller for the following input tuples: $(\xi_1, \xi_2) \in \{(0,0), (0.4, 0.5), (-0.7, 0.9), (-0.5, 0)\}$.

b) Determine crisp output values from the fuzzy outputs computed in part a) using the mean of maxima method and the center of gravity method (for COG an approximation is sufficient, you need not do the exact calculations, which are tedious in some cases.)

Assignment 33 Mamdani-Assilian Controller

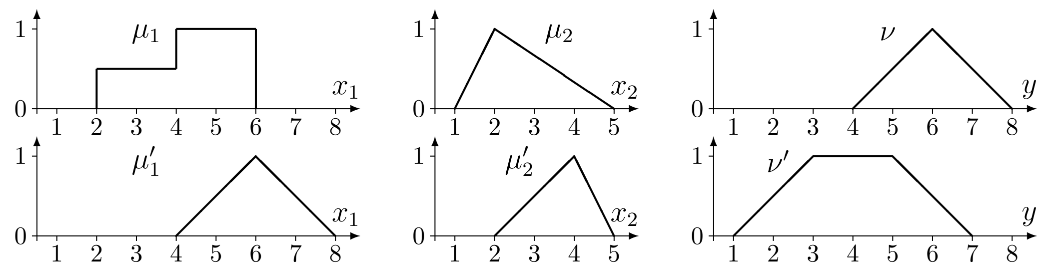
Design a Mamdani-Assilian controller with two inputs $\xi_1 \in X_1 = [0,3]$ and $\xi_2 \in X_2 = [0,3]$ and one output $\eta \in Y = [0,4]$, which uses center of gravity as the defuzzication method. This fuzzy controller should compute the following mappings:

$$(1,0) \mapsto 2, \quad (1,3) \mapsto 4,$$
$$(0,2) \mapsto 2, \quad (2,2) \mapsto 4,$$
$$(2,0) \mapsto 2.$$

Try to use as few fuzzy sets as possible. Determine the output of your fuzzy controller for the two input tuples $(1.5, 1.5)$ and $(0.5, 1.5)$.

Assignment 34 Mamdani-Assilian Controller

Let the following fuzzy sets and rules be given:



$$R_1: \text{ if } x_1 \text{ is } \mu_1 \text{ and } x_2 \text{ is } \mu_2 \text{ then } y \text{ is } \nu$$
$$R_2: \text{ if } x_1 \text{ is } \mu_1' \text{ and } x_2 \text{ is } \mu_2' \text{ then } y \text{ is } \nu'$$

a) Based on these fuzzy sets and the rules, which output μ_{output} does a Mamdani-Assilian controller return for the input tuple $(5, 2.5)$?

b) Which crisp output values are (approximately) obtained by defuzzification of the output set with both mean of maxima method and center of gravity method?