Fuzzy Systems
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Assignment Sheet 6

Assignment 20 Fuzzy Relations

Let the fuzzy relation R be defined on the sets $X_1 = \{a, b, c\}$, $X_2 = \{s, t\}$, $X_3 = \{x, y\}$ and $X_4 = \{i, j\}$. Furthermore, let R be different than 0 at the following positions:

$$\begin{aligned} R(a, t, y, j) &= 0.2, \\ R(b, s, x, j) &= 0.5, \\ R(a, s, y, j) &= 1.0, \\ R(a, s, y, i) &= 0.9, \\ R(b, t, y, i) &= 0.7, \\ R(c, s, y, j) &= 0.3. \end{aligned}$$

a) Compute the following projections of R:

$$\begin{aligned} R_{1,2,4} &= [R \downarrow \{X_1, X_2, X_4\}], \\ R_{1,3} &= [R \downarrow \{X_1, X_3\}], \\ R_4 &= [R \downarrow \{X_4\}]. \end{aligned}$$

b) Compute the following cylindric extensions:

$$\begin{aligned} &[R_{1,2,4} \uparrow \{X_3\}], \\ &[R_{1,3} \uparrow \{X_2, X_4\}], \\ &[R_4 \uparrow \{X_1, X_2, X_3\}]. \end{aligned}$$

Assignment 21 Fuzzy Relations

Prove that not every fuzzy relation R on $X \times Y$ is the Cartesian product of two fuzzy sets A of X and B of Y.

Assignment 22 Fuzzy Relations

Let R be a fuzzy relation on $X \times Y$ and S, T fuzzy relations on $Y \times Z$. Find an example where $R \circ (S \cap T) \subset (R \circ S) \cap (R \circ T)$ holds.

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Assignment 23 Fuzzy Binary Relations

The fuzzy binary relation R is defined on set $X = \{1, 2, \ldots, 100\}$ and $Y = \{50, 51, \ldots, 100\}$ and represents the relation "x is much smaller than y". It is defined by its membership function

$$R(x, y) = \begin{cases} 1 - \frac{x}{y}, & \text{if } x \leq y \\ 0, & \text{otherwise,} \end{cases}$$

whereas $x \in X$ and $y \in Y$.

a) What is the domain of R?

b) What is the range of R?

c) What is the height of R?

d) Calculate R^{-1}.