Assignment Sheet 6

Assignment 20  Fuzzy Relations

Let the fuzzy relation \( R \) be defined on the sets \( X_1 = \{a, b, c\} \), \( X_2 = \{s, t\} \), \( X_3 = \{x, y\} \) and \( X_4 = \{i, j\} \). Furthermore, let \( R \) be different than 0 at the following positions:

\[
R(a, t, y, j) = 0.2, \\
R(b, s, x, j) = 0.5, \\
R(a, s, y, j) = 1.0, \\
R(a, s, y, i) = 0.9, \\
R(b, t, y, i) = 0.7, \\
R(c, s, y, j) = 0.3.
\]

a) Compute the following projections of \( R \):

\[
R_{1,2,4} = [R \downarrow \{X_1, X_2, X_4\}], \\
R_{1,3} = [R \downarrow \{X_1, X_3\}], \\
R_4 = [R \downarrow \{X_4\}].
\]

b) Compute the following cylindric extensions:

\[
[R_{1,2,4} \uparrow \{X_3\}], \\
[R_{1,3} \uparrow \{X_2, X_4\}], \\
[R_4 \uparrow \{X_1, X_2, X_3\}].
\]

Assignment 21  Fuzzy Relations

Prove that not every fuzzy relation \( R \) on \( X \times Y \) is the Cartesian product of two fuzzy sets \( A \) of \( X \) and \( B \) of \( Y \).

Assignment 22  Fuzzy Relations

Let \( R \) be a fuzzy relation on \( X \times Y \) and \( S, T \) fuzzy relations on \( Y \times Z \). Find an example where \( R \circ (S \cap T) \subset (R \circ S) \cap (R \circ T) \) holds.
The fuzzy binary relation $R$ is defined on set $X = \{1, 2, \ldots, 100\}$ and $Y = \{50, 51, \ldots, 100\}$ and represents the relation “$x$ is much smaller than $y$”. It is defined by its membership function

$$R(x, y) = \begin{cases} 1 - \frac{x}{y}, & \text{if } x \leq y \\ 0, & \text{otherwise} \end{cases},$$

whereas $x \in X$ and $y \in Y$.

a) What is the domain of $R$?

b) What is the range of $R$?

c) What is the height of $R$?

d) Calculate $R^{-1}$. 