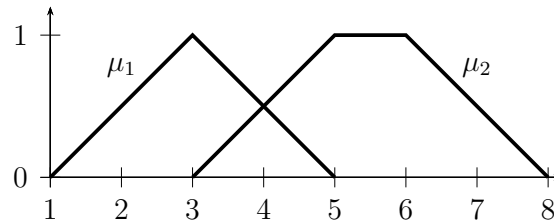


Assignment Sheet 4

Assignment 12 Fuzzy Set Operations

Let the following two fuzzy sets be given:



Compute and draw for each of the pairs

- a) the complement of μ_1 w.r.t. $U = [1, 8]$ using the standard fuzzy negation,
- b) the intersection of μ_1 and μ_2 using the standard fuzzy t -norm \top_{\min} ,
- c) the intersection of μ_1 and μ_2 using the algebraic product \top_{prod} ,
- d) the intersection of μ_1 and μ_2 using the Łukasiewicz t -norm $\top_{\text{Łuka}}$,
- e) the union of μ_1 and μ_2 using the standard fuzzy t -conorm \perp_{\max} ,
- f) the union of μ_1 and μ_2 using the algebraic sum \perp_{sum} ,
- g) the union of μ_1 and μ_2 using the Łukasiewicz t -conorm $\perp_{\text{Łuka}}$.

Assignment 13 Fuzzy Negation

In order to construct an involutive negation, one can use either a strictly monotonously increasing or decreasing generator function:

Theorem: $\sim: [0, 1] \mapsto [0, 1]$ is an involutive fuzzy negation if there exists a continuous function $g: [0, 1] \mapsto \mathbb{R}$ that fulfills the following properties:

- (i) $g(0) = 0$.
- (ii) g is strictly monotonously increasing.
- (iii) $\sim a = g^{-1}(g(1) - g(a))$.

Theorem: $\sim: [0, 1] \mapsto [0, 1]$ is an involutive fuzzy negation if there exists a continuous function $f: [0, 1] \mapsto \mathbb{R}$ that fulfills the following properties:

- (i) $f(1) = 0$.
- (ii) f is strictly monotonously decreasing.
- (iii) $\sim a = f^{-1}(f(0) - f(a))$.

Fuzzy Systems

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Now, consider the class of increasing generator functions

$$g_\lambda(a) = \frac{a}{\lambda + (1 - \lambda)a}.$$

Apply the given theorem, which allows to construct an involutive fuzzy negation from an arbitrary continuous and strictly increasing function g with $g(0) = 0$. Draw the resulting fuzzy negation for several values of λ .

Assignment 14 Fuzzy Conjunction

Prove the following theorem which was given in the lecture:

Theorem: For all t -norms \top and all fuzzy truth values $a, b \in [0, 1]$ it is

$$\top_{-1}(a, b) \leq \top(a, b) \leq \top_{\min}(a, b),$$

where $\top_{\min}(a, b) = \min\{a, b\}$ is the standard fuzzy conjunction and \top_{-1} is the so-called drastic product

$$\top_{-1}(a, b) = \begin{cases} a & \text{if } b = 1, \\ b & \text{if } a = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Assignment 15 Fuzzy Disjunction

Consider the class of increasing generator functions (cf. Assignment 13)

$$g_\lambda(a) = \frac{a}{\lambda + (1 - \lambda)a}.$$

Apply the theorem of the lecture which allows to construct a fuzzy disjunction (t -conorm) from an arbitrary continuous and strictly increasing function g with $g(0) = 0$. If you have a proper software tool like, for instance, **gnuplot** available, plot the resulting fuzzy disjunction for several values of λ .