Chapter 7:

Self-Organizing Maps

Self-Organizing Maps

A **self-organizing map** or **Kohonen feature map** is a neural network with a graph $G = (U, C)$ that satisfies the following conditions

(i) $U_{\text{hidden}} = \emptyset$, $U_{\text{in}} \cap U_{\text{out}} = \emptyset$,

(ii) $C = U_{\text{in}} \times U_{\text{out}}$.

The network input function of each output neuron is a **distance function** of input and weight vector. The activation function of each output neuron is a **radial function**, i.e. a monotonously decreasing function

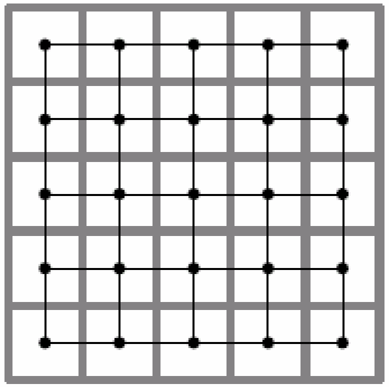
$$f : \mathbb{R}_0^+ \to [0, 1] \quad \text{with} \quad f(0) = 1 \quad \text{and} \quad \lim_{x \to \infty} f(x) = 0.$$

The output function of each output neuron is the identity.
The output is often discretized according to the "**winner takes all**" principle.
On the output neurons a **neighborhood relationship** is defined:

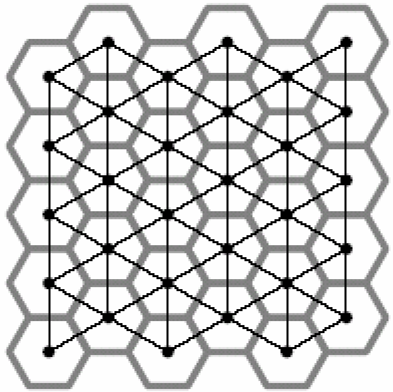
$$d_{\text{neurons}} : U_{\text{out}} \times U_{\text{out}} \to \mathbb{R}_0^+ .$$

Self-Organizing Maps: Neighborhood

Neighborhood of the output neurons: neurons form a grid



quadratic grid

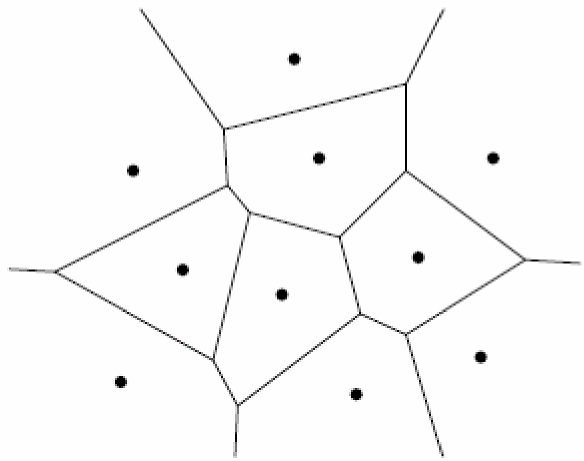


hexagonal grid

- Thin black lines: Indicate nearest neighbors of a neuron.
- Thick gray lines: Indicate regions assigned to a neuron for visualization.

Vector Quantization

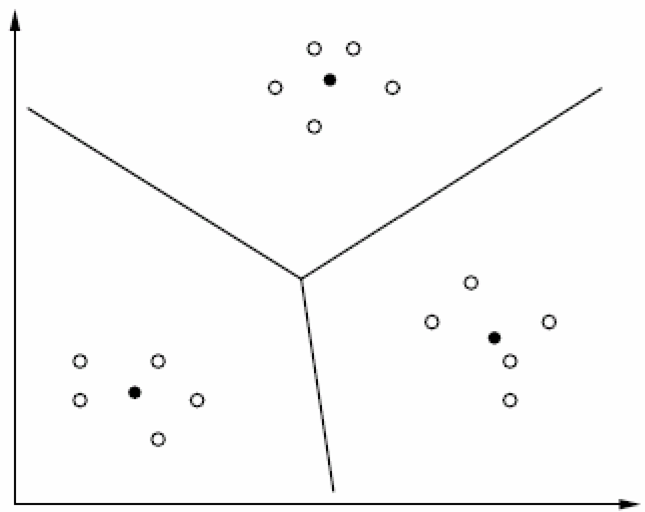
Voronoi diagram of a vector quantization



- Dots represent vectors that are used for quantizing the area.
- Lines are the boundaries of the regions of points that are closest to the enclosed vector.

Learning Vector Quantization

Finding clusters in a given set of data points



- Data points are represented by empty circles (○).
- Cluster centers are represented by full circles (●).

Learning Vector Quantization

Adaptation of reference vectors / codebook vectors

- For each training pattern find the closest reference vector.
- Adapt only this reference vector (winner neuron).
- For classified data the class may be taken into account.
  (reference vectors are assigned to classes)

Attraction rule (data point and reference vector have same class)

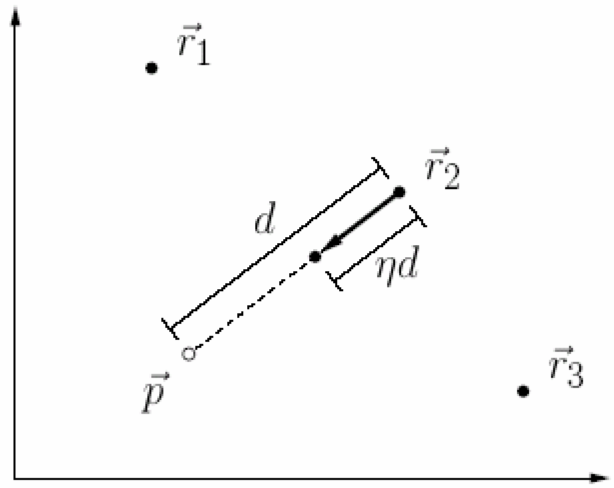
$$\vec{r}^{\,(\text{new})} = \vec{r}^{\,(\text{old})} + \eta(\vec{p} - \vec{r}^{\,(\text{old})}),$$

Repulsion rule (data point and reference vector have different class)

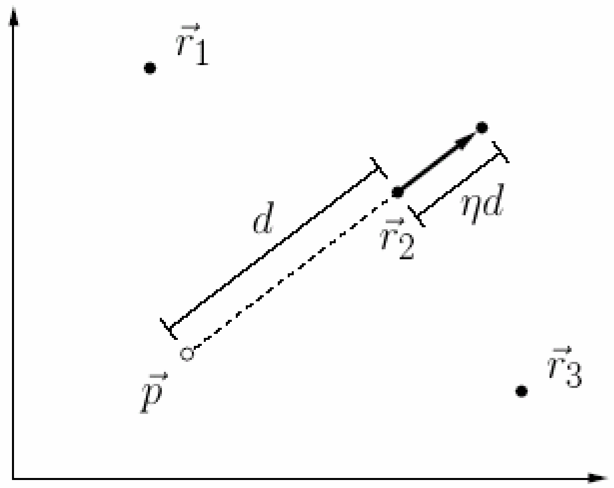
$$\vec{r}^{\,(\text{new})} = \vec{r}^{\,(\text{old})} - \eta(\vec{p} - \vec{r}^{\,(\text{old})}).$$

Learning Vector Quantization

Adaptation of reference vectors / codebook vectors



attraction rule

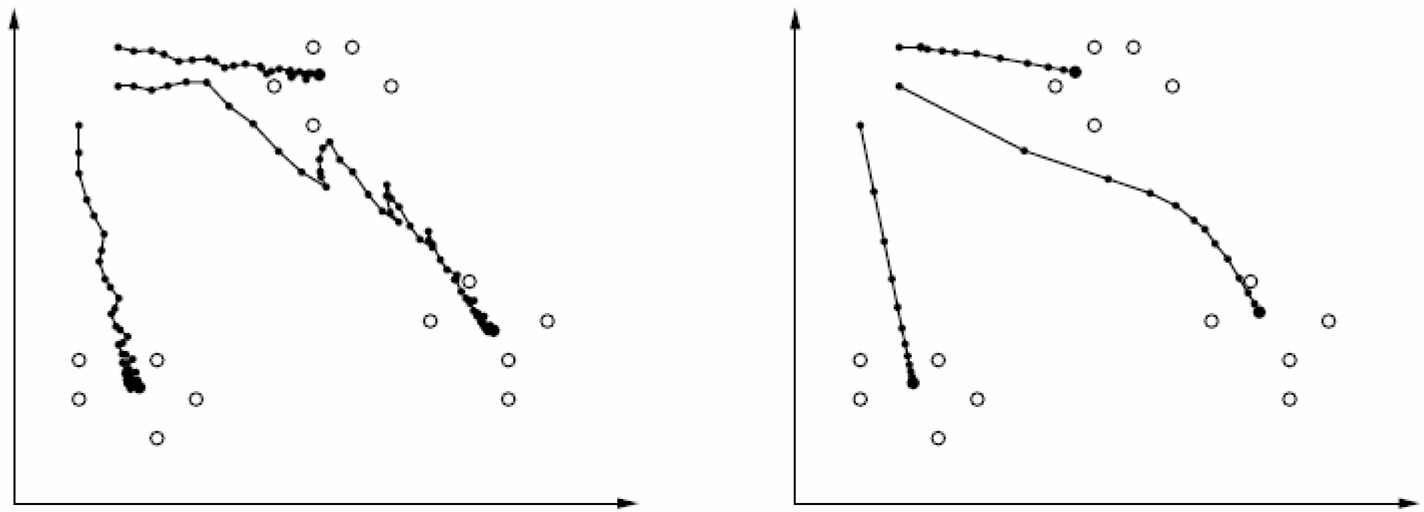


repulsion rule

- $\vec{p}$: data point, $\vec{r}_i$: reference vector
- $\eta = 0.4$ (learning rate)

Learning Vector Quantization: Example

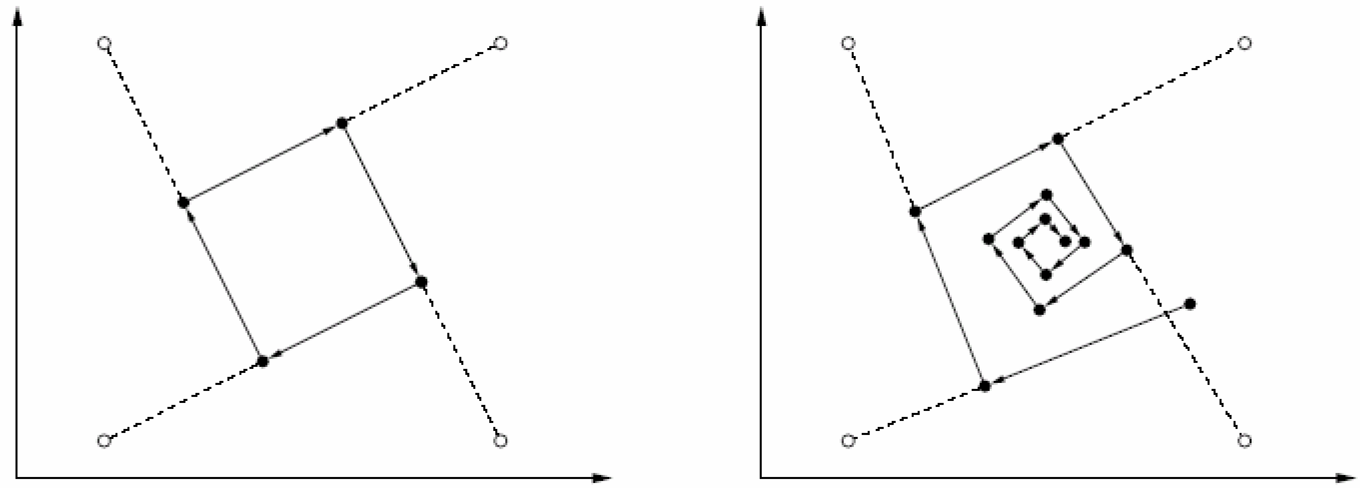
Adaptation of reference vectors / codebook vectors



- Left: Online training with learning rate $\eta = 0.1$,
- Right: Batch training with learning rate $\eta = 0.05$.

Learning Vector Quantization: Learning Rate Decay

Problem: fixed learning rate can lead to oscillations



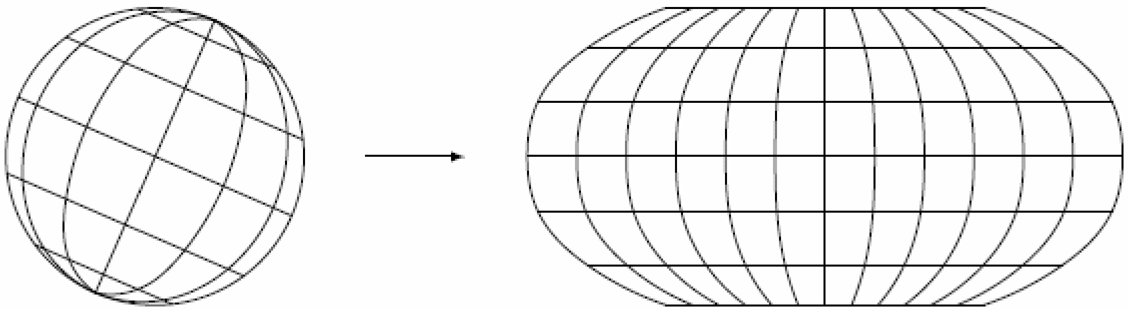
Solution: time dependent learning rate

$$\eta(t) = \eta_0 \alpha^t, \quad 0 < \alpha < 1, \qquad \text{or} \qquad \eta(t) = \eta_0 t^{\kappa}, \quad \kappa > 0.$$

Topology Preserving Mapping

Images of points close to each other in the original space should be close to each other in the image space.

Example: **Robinson projection** of the surface of a sphere



- Robinson projection is frequently used for world maps.

Self-Organizing Maps: Neighborhood

Find topology preserving mapping by respecting the neighborhood

Reference vector update rule:

$$\vec{r}_u^{\,(\text{new})} = \vec{r}_u^{\,(\text{old})} + \eta(t) \cdot f_{\text{nb}}(d_{\text{neurons}}(u, u_*), \varrho(t)) \cdot (\vec{p} - \vec{r}_u^{\,(\text{old})}),$$

- u_* is the winner neuron (reference vector closest to data point).
- The function f_{nb} is a radial function.

Time dependent learning rate

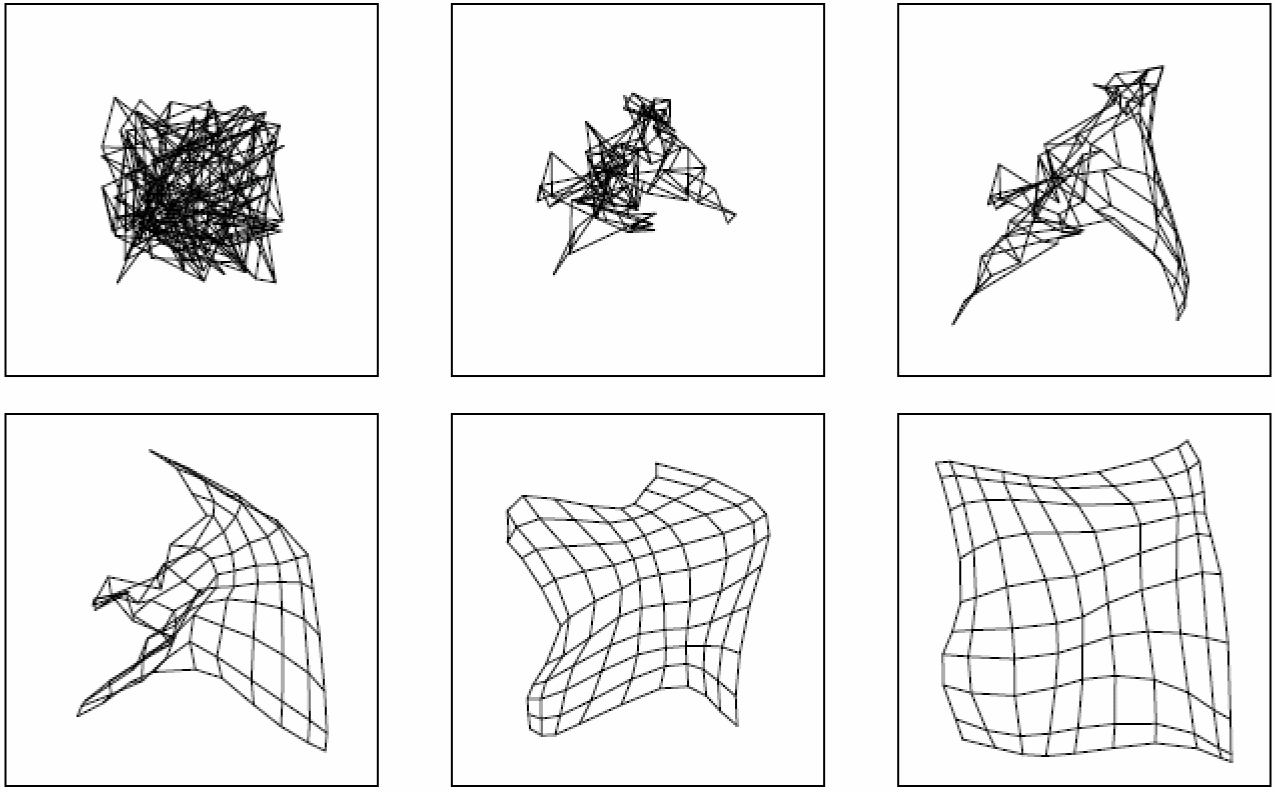
$$\eta(t) = \eta_0 \alpha_\eta^t, \quad 0 < \alpha_\eta < 1, \qquad \text{or} \qquad \eta(t) = \eta_0 t^{\kappa_\eta}, \quad \kappa_\eta > 0.$$

Time dependent neighborhood radius

$$\varrho(t) = \varrho_0 \alpha_\varrho^t, \quad 0 < \alpha_\varrho < 1, \qquad \text{or} \qquad \varrho(t) = \varrho_0 t^{\kappa_\varrho}, \quad \kappa_\varrho > 0.$$

Self-Organizing Maps: Examples

Example: Unfolding of a two-dimensional self-organizing map.



Self-Organizing Maps: Examples

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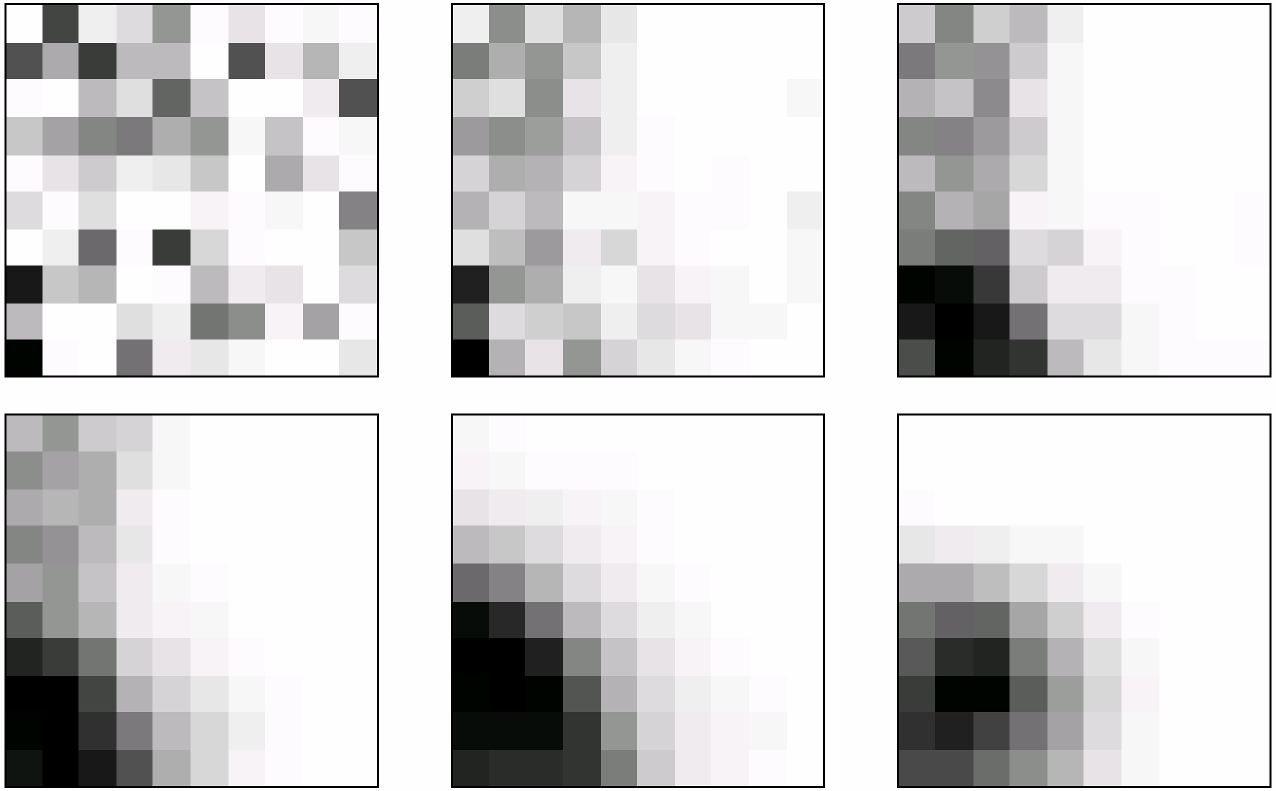

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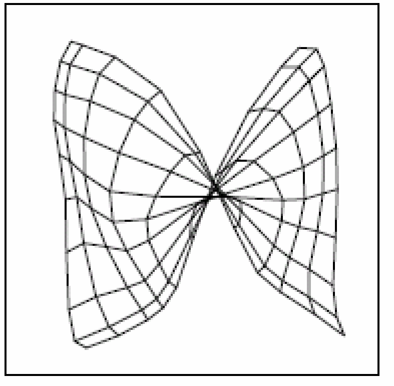

Training a self-organizing map may fail if

- the (initial) learning rate is chosen too small or
- or the (initial) neighbor is chosen too small.

Self-Organizing Maps: Examples

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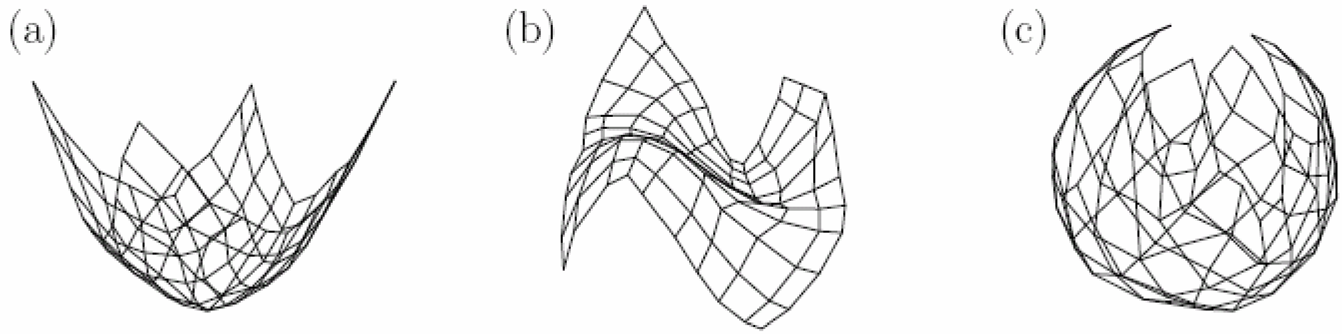

Self-organizing maps that have been trained with random points from
(a) a rotation parabola, (b) a simple cubic function, (c) the surface of a sphere.

- In this case original space and image space have different dimensionality.
- Self-organizing maps can be used for dimensionality reduction.