## Exercise Sheet 11

## Exercise 35 Fuzzy Clustering

Consider the objective function of fuzzy clustering with a fuzzifier $w=1$, that is,

$$
J(\mathbf{X}, \mathbf{B}, \mathbf{U})=\sum_{i=1}^{c} \sum_{j=1}^{n} u_{i j} d^{2}\left(\beta_{i}, \vec{x}_{j}\right),
$$

which is to be minimized under the constraint

$$
\forall j \in\{1, \ldots, n\}: \quad \sum_{i=1}^{c} u_{i j}=1
$$

Show that one obtains a hard/crisp assignment of the data points even if the membership degrees $u_{i j}$ may come from the interval $[0,1]$. That is, show that for the minimum of the objective function $J$ it is $\forall i \in\{1, \ldots, c\}: \forall j \in\{1, \ldots, n\}: u_{i j} \in\{0,1\}$.
(Hint: You may find it easier to consider the special case $c=2$ (two clusters) and to examine the term for a single data point $\vec{x}_{j}$. Then generalize the result.)

Exercise 36 Agglomerative Clustering
Let the following one-dimensional data set be given:

$$
2,5,11,12,17,21,32 .
$$

Process this data set with hierarchical agglomerative clustering using
a) the centroid method,
b) the single linkage methode,
c) the complete linkage methode!

Draw a dendrogram for each case!

Exercise 37 Method of Least Squares/Regression
Determine a best fit line $y=a+b x$ (regression line) for the data set already considered in exercise 10, that is, for

| $x$ | 0 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ | 0 | 1 | 2 | 3 | 2 | 3 | 4 | 4 | 6 | 5 |

a) using the covariance and the variances/standard deviations (see the lecture slides, section on correlation coefficients)
b) using the method of least squares/the system of normal equations!

Draw a diagram of the data points and the regression line!

## Exercise 38 Logistic Regression

The following table shows the number of American intercontinental ballistic missiles (ICBMs) in the years from 1960 to 1969:

| year, $x$ | 1960 | 1961 | 1962 | 1963 | 1964 | 1965 | 1966 | 1967 | 1968 | 1969 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| number, $y$ | 18 | 63 | 294 | 424 | 834 | 854 | 904 | 1054 | 1054 | 1054 |

Find a best fit curve for this data set using logistic regression $(Y=1060)$ ! Draw the original data and sketch the curve $y=\frac{1060}{1+e^{a+b x}}$ !

## Exercise 39 Frequent Item Set Mining

Given the transactions from the table on the right determine all frequent item sets with a minimum support $s_{\text {min }}=3$ using the apriori algorithm. Which item sets are closed, which are maximal?

| $t_{I D}$ | items |
| :--- | :--- |
| $1:$ | $\{\mathrm{a}, \mathrm{d}, \mathrm{e}\}$ |
| $2:$ | $\{\mathrm{b}, \mathrm{c}, \mathrm{d}\}$ |
| $3:$ | $\{\mathrm{a}, \mathrm{c}, \mathrm{d}\}$ |
| $4:$ | $\{\mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}\}$ |
| $5:$ | $\{\mathrm{a}, \mathrm{d}\}$ |
| $6:$ | $\{\mathrm{b}, \mathrm{c}, \mathrm{d}\}$ |
| $7:$ | $\{\mathrm{a}, \mathrm{b}, \mathrm{e}\}$ |
| $8:$ | $\{\mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}\}$ |
| $9:$ | $\{\mathrm{a}, \mathrm{d}, \mathrm{e}\}$ |
| $10:$ | $\{\mathrm{a}, \mathrm{b}, \mathrm{d}\}$ |
| $11:$ | $\{\mathrm{c}, \mathrm{d}\}$ |

