## Exercise Sheet 10

## Exercise $35 \quad c$-Means Clustering

Consider the following two-dimensional data set:

| $x$ | 1 | 6 | 8 | 3 | 2 | 2 | 6 | 6 | 7 | 7 | 8 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ | 5 | 2 | 1 | 5 | 4 | 6 | 1 | 8 | 3 | 6 | 3 | 7 |

Process this data set with $c$-means clustering with $c=3$ (i.e., try to find 3 clusters)! Use the first three data tuples als initial positions for the cluster centers and observe the migration of the centers.

## Exercise $36 \quad c$-Means Clustering

In exercises 25 and 26 on sheet 7 we considered a simple two-dimensional data set. Reconsider this data set, but assume that that no class information is available for the data points. That is, consider the following data set:

| $x$ | 3 | 3 | 4 | 4 | 5 | 6 | 7 | 7 | 8 | 9 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 6 | 5 | 7 | 3 | 4 | 5 | 6 | 6 | 7 | 8 | 8 | 8 | 9 |

a) Which problem of $c$-means clustering becomes obvious when this data set is processed with $c=2$ (i.e., if one tries to find two clusters)?
Hint: What is the desired result? What is produced by c-means clustering?
(You need not compute the exact result of the algorithm, a qualitative description suffices. Compare the result to a naive Bayes classifier.)
b) How could one try to cope with this problem?

Hint: Recall what distinguishes a full and a naive Bayes classifier.

## Exercise 37 Fuzzy Clustering

Consider the one-dimensional data set

$$
1,3,4,5,8,10,11,12 .
$$

We want to process this data set with fuzzy $c$-means clustering with $c=2$ (two clusters) and a fuzzifier of $w=2$. Assume that the cluster centers are initialized to 1 and 5 . Execute one step of alternating optimization as it is used for fuzzy clustering, i.e.:
a) Compute the membership degrees of the data points for the initial cluster centers!
b) Compute new cluster centers from the membership degrees computed in this way!

## Exercise 38 Expectation Maximization

Consider again the one-dimensional data set used in exercise 37, which we want to process in this exercise with the expectation maximization algorithm to estimate the parameters of a mixture of two normal/Gaussian distributions. Let the prior probabilities of the two clusters be fixed to $\theta_{i}=\frac{1}{2}$ and the variances to $\sigma_{i}^{2}=1, i=1,2$. (That is, only the expected values of the normal distributions - the cluster centers - are to be adapted.) Use the same values for the initial cluster centers as in exercise 40, that is, 1 and 5 . Compute one expectation step and one maximization step, i.e.:
a) Compute the posterior probabilities of the data points for the initial cluster centers!
b) Estimate new cluster centers from the data point weights computed in this way!

