Example 7.1

Fuzzy Control Rules:

a) If temperature is very high and the pressure is slightly low
   then the heat change should be slightly negative.

b) If rate of descent = positive big, airspeed = negative big,
   and glide slope = positive big
   then rpm change = positive big and elevator angle change = insignificant change.

Theorem 7.2

a) Let “if A then B” be a rule with $\mu_A \in F(X)$ and $\mu_B \in F(Y)$.
   Then the relational equation $B=A \bullet R$ can be solved iff the Gödel relation $A \ominus B$ is a solution.

\[ \mu_{A \ominus B}: X \times Y \rightarrow [0,1] \]
\[ \mu_{A \ominus B}(x,y) = \begin{cases} 1, & \text{if } \mu_A(x) \leq \mu_B(y) \text{ and } \\ \mu_B(y), & \text{otherwise.} \end{cases} \]

b) If $R$ with $B=A \bullet R$ is a solution, then the set of solutions $R=\{\mu_S \in F(X \times Y) | B=A \bullet S\}$
   has the following properties: if $\mu_S', \mu_S'' \in R$ then $\mu_{S' \cup S''} \in R$
c) If $A \ominus B$ is a solution, then $A \ominus B$ is the largest solution with respect to $\subseteq$.

**Theorem 7.3**

Let $R$ be a fuzzy relation with $B_i = A_i \circ R$ for $i=1\ldots n$.

a) If there is a solution for the system, then the set of solutions is a upper semi lattice.

b) There is a solution, iff $\bigcap_{i=1}^{n} A_i \ominus B_i$ is a solution.

c) If $\bigcap_{i=1}^{n} A_i \ominus B_i$ is a solution, then this solution is the biggest solution w.r.t. to inclusion.

**Remark 7.4**

If there is no solution, the Gödelrelation is a good approximation.
Example 7.5

a) Imprecise rule:
   if \( x \in [2,3] \) the \( y \in [5,6] \)

b) Set of imprecise rules:
   if \( A_i \) then \( B_i \), \( i=1,2,3 \)

c) Conclusion
d) Fuzzy rule: 
If $x \sim A$ then $y \sim B$

Layer 1

Layer 0.75

Layer 0.5

Layer 0.25

e) Set of fuzzy rules: 
If $x = A_i$ then $y = B_i$ \quad i=1,2,...