# 7. Fuzzy Control Based on Relational Equations

## Example 7.1

Fuzzy Control Rules:

a) **If** temperature is very high and the pressure is slightly low
**then** the heat change should be slightly negative.

b) **If** rate of descent = positive big, airspeed = negative big.
and glide slope = positive big
**then** rpm change = positive big and elevator angle change = insignificant change.

## Theorem 7.2

a) Let "if A then B" be a rule with $\mu_A \in F(X)$ and $\mu_B \in F(Y)$.
Then the relational equation $B=A \bullet R$ can be solved iff the Gödel relation $A \ominus B$ is a solution.

$\mu_{A \ominus B}: X \times Y \to [0,1]$ is defined by

$$\mu_{A \ominus B}(x,y) = \begin{cases} 1, & \text{if } \mu_A(x) \le \mu_B(y) \text{ and} \\ \mu_B(y), & \text{otherwise.} \end{cases}$$

b) If R with $B=A \bullet R$ is a solution, then the set of solutions $R=\{\mu_S \in F(X \times Y) | B=A \bullet S\}$ has the following properties: if $\mu_{S'}, \mu_{S''} \in R$ then $\mu_{S' \cup S''} \in R$

c) If $A \odot B$ is a solution, then $A \odot B$ is the largest solution with respect to $\subseteq$.

## Theorem 7.3

Let R be a fuzzy relation with $B_i = A_i \bullet R$ for i=1…n.

a) If there is a solution for the system, then the set of solutions is a upper semi lattice.

b) There is a solution, iff $\bigcap_{i=1}^{n} A_i \odot B_i$ is a solution.

c) If $\bigcap_{i=1}^{n} A_i \odot B_i$ is a solution, then this solution is the biggest solution w.r.t. to inclusion.

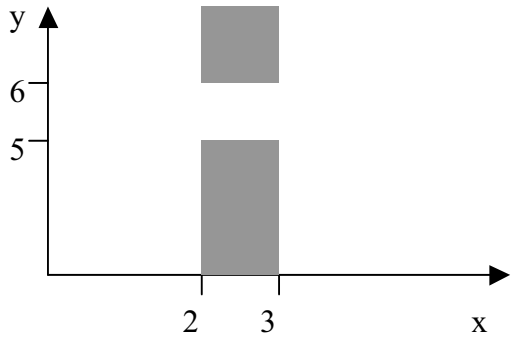
## Remark 7.4

If there is no solution, the Gödelrelation is a good approximation.

## Example 7.5

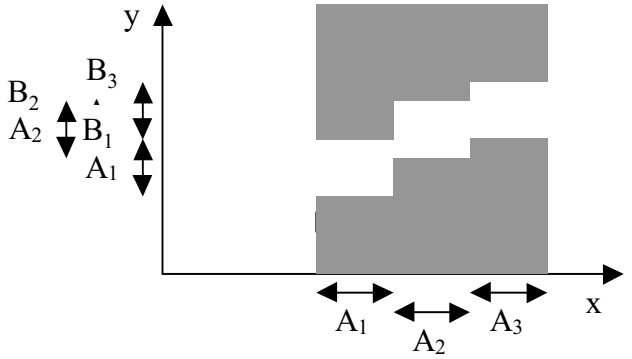
a) Imprecise rule:

if x ∈ [2,3] the y ∈ [5,6]

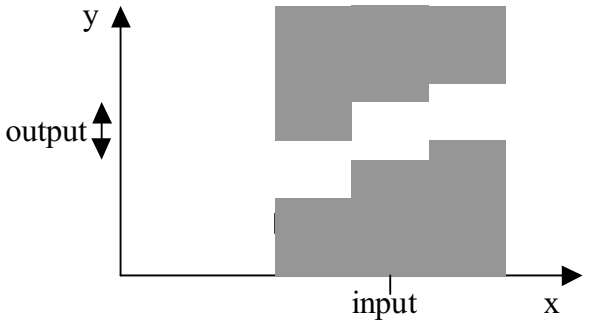



b) Set of imprecise rules:

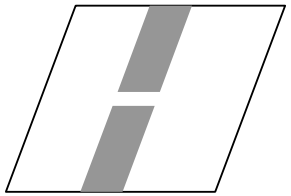
if $A_i$ then $B_i$, i=1,2,3



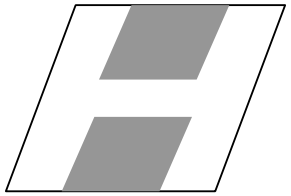

c) Conclusion



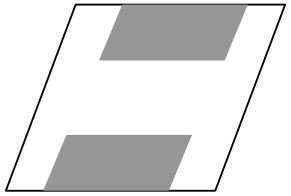
d) Fuzzy rule:
If $x=\underset{\sim}{A}$ then $y=\underset{\sim}{B}$



Layer 1



Layer 0,75



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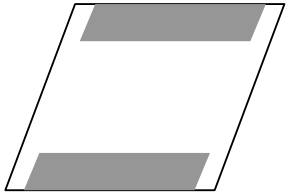
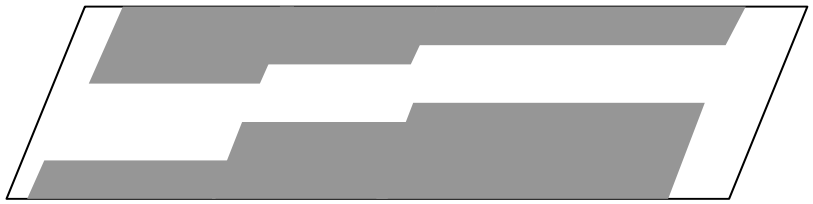


Layer 0,25

e) Set of fuzzy rules:
If $x=\underset{\sim}{A}_i$ then $y=\underset{\sim}{B}_i$ i=1,2,...



etc.