4. The Extension Principle

How to extend a mapping of the form $\Phi : X^n \to Y$ to a mapping of the kind $\hat{\Phi} : F(X)^n \to F(Y)$?

Let P be a set of vague statements, which can be combined by the statements *and* and *or*. The mapping acc:$P \to [0,1]$ may assign an acceptance degree acc(a) to every statement a$\in P$. acc(a)=0 means that a is definitely false, acc(a)=1, on the other hand, that it is definitely true. However, if acc(a)$\in$(0,1), then we can only speak of a *gradual truth* of the statement a.

If we combine two statements a,b$\in P$, their combination is rated according to the following scheme:

acc(a and b) = acc (a$\wedge$b) = min{acc(a), acc(a)}

acc(a or b) = acc (a$\vee$b) = max{acc(a), acc(a)}

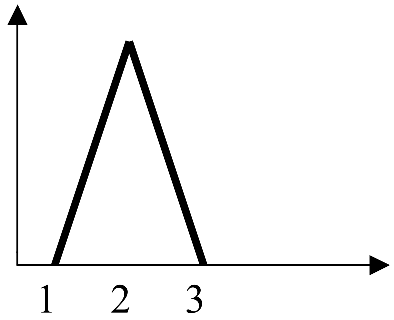
Remark 4.1

We define

acc(„for all i$\in$I statement a_i holds“) = inf{acc(a_i)|i$\in$I}

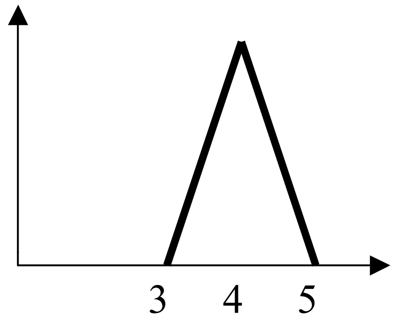
acc(„there is a i$\in$I such that a_i holds“)= sup{acc(a_i) | i$\in$I}

Example 4.2

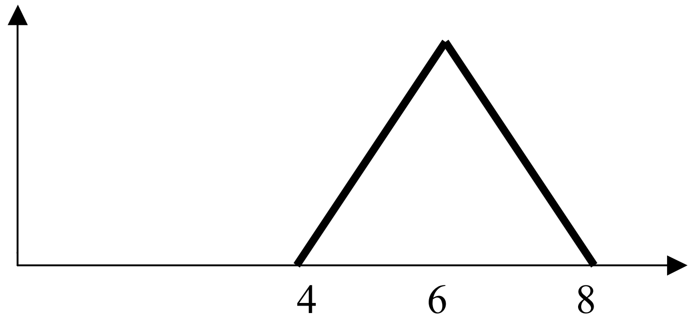


approximately 2

+



approximately 4



approximately 2 + approximately 4

Example 4.3

Let X,Y be sets. ϕ:X$\rightarrow$Y

How to extend ϕ to $\hat{\Phi}: F(X)^n \rightarrow F(Y)$?

Def. 4.4

Let ϕ:$X^n \rightarrow$Y be a mapping. The extension of ϕ is given by

$\hat{\Phi}: F(X)^n \rightarrow F(Y)$ with

$$\hat{\Phi}(\mu_1,\ldots,\mu_n)(y) = \sup\{\min\{\mu_1(x_1),\ldots,\mu_n(x_n)\} \mid \Phi(x_1,\ldots,x_n) = y\}$$

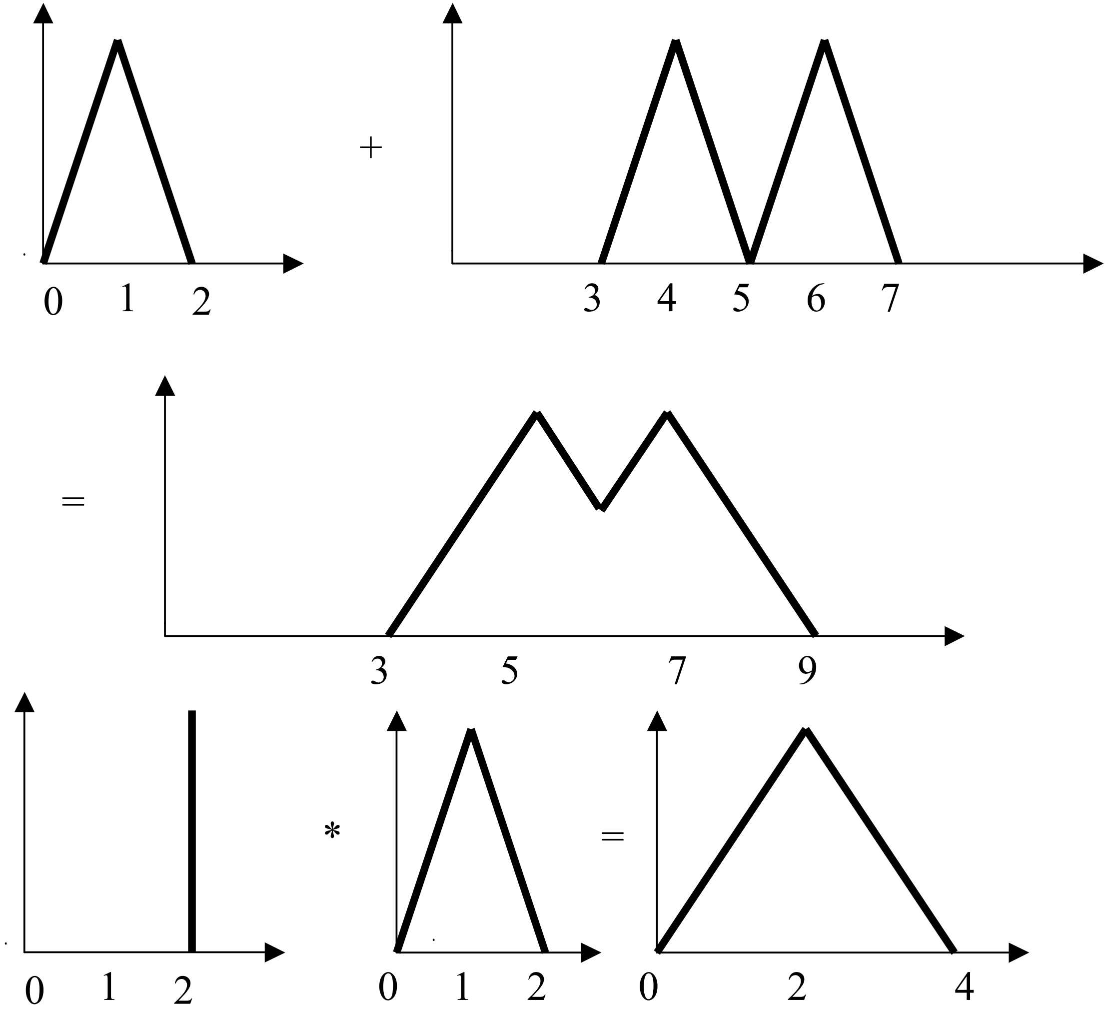
Def. 4.5 (arithmetic functions)

a) $(\mu_1 + \mu_2)(t) := \sup_{x,y \in \Re : x+y=t} \{\min(\mu_1(x), \mu_2(y))\}$

b) $(a \cdot \mu_1)(t) := \sup_{x \in \Re : a \cdot x = t} \{\min(\mu_1(x))\}$

Other functions are defined in the same way.

Example 4.6



Theorem 4.6

a) μ_1, μ_2 fuzzy numbers $\Rightarrow$ $\mu_1+\mu_2$, $\mu_1*\mu_2$, and $\mu_1-\mu_2$ are fuzzy numbers.

b) μ_1, μ_2 fuzzy numbers $\not\Rightarrow$ μ_1/μ_2 fuzzy numbers.

c) $\mu_1+I_{\{0\}}=\mu_1$

d) $\mu_1*I_{\{1\}}=\mu_1$

Theorem 4.7 (Minkowski operations)

a) $\mu_\alpha \oplus \mu'_\alpha = (\mu+\mu')_\alpha \quad (\alpha > 0)$

b) $\{a\}\otimes\mu_\alpha = (a\mu)_\alpha \ (\alpha > 0)$

where $A\oplus B=\{x+y|x\in A, y\in B\}$

$A\otimes B=\{x*y|x\in A, y\in B\}$