11. Sugeno - Takagi Controller

Rules: \( L_1, \ldots, L_n \)

\[ L_i : \text{if } x_1 \text{ is } \mu_{i,1} \text{ and } x_2 \text{ is } \mu_{i,2} \text{ and } \ldots \text{ and } x_p \text{ is } \mu_{i,p} \]

\[ \text{then } y_i = c_{0,i} + c_{1,i} x_1 + \ldots + c_{p,i} x_p \]

Matching degree for input \((x_1^0, \ldots, x_p^0)\) and rule \(L_i\)

\[ w_i = \mu_{i,1}(x_1^0) \land \ldots \land \mu_{i,p}(x_p^0) \]

Output for \((x_1^0, x_p^0)\)

\[ y^0 = \sum_{i=1}^{p} w_i \cdot y_i^0 \bigg/ \sum_{i=1}^{p} w_i \quad , \quad y_i^0 = c_{0,i} + \ldots + c_{p,i} x_p^0 \]
Examples

L₁ : if $x₁$ is 3, then $y₁ = 1 \cdot x₁ + 0.5 \cdot x₂ + 1$

L₂ : if $x₁$ is and $x₂$ is 3, then $y₂ = -0.1x₁ + 4x₂ + 1.2$

L₃ : if $x₁$ is and $x₂$ is 3, then $y₃ = 0.9x₁ + 0.7x₂ + 9$

L₄ : if $x₁$ is and $x₂$ is 4, then $y₄ = 0.2x₁ + 0.1x₂ + 0.2$

$(x₁^0, x₂^0) = (6, 7)$, $y^0 = 19.5$
Example 9.1 Computation of several fuzzy rules

**Antecedents**

R1:

*If* $x_1$ is high and $x_2$ is low then $y = f_2(x_1, y_1)$

R2:

*If* $x_1$ is middle and $x_2$ is middle then $y = f_1(x_1, y_1)$

*If* $x_1$ is high and $x_2$ is low then $y = f_2(x_1, y_1)$

Crisp input: $x_1$ and $x_2$

**Consequents**

Decision logic output:

$$y = \frac{\alpha_1 f_1(x_1, y_1) + \alpha_2 f_2(x_1, y_1)}{\alpha_1 + \alpha_2}$$
Definition 11.2  Sugeno-Takagi Fuzzy Control

A Sugeno Fuzzy Controller consists of a set of rules $R_i$, $i=1,...,k$:

$R_i$: if $x_1$ is $A_{i1}$ and if $x_2$ is $A_{i2}$ and ... and if $x_n$ is $A_{in}$
then $y=f_i(x_1,x_2,...,x_n)$

where $A_{ij}$ are fuzzy sets and $f_i(x_1,x_2,...,x_n)$ is linear.

\[ f_i(x_1,x_2,...,x_n) = a_1x_1 + a_2x_2 + ... + a_nx_n + a_{n+1} \]

The output is computed by

\[ y = \frac{\sum_{i=1}^{k} \alpha_i f_i(x_1,...,x_n)}{\sum_{i=1}^{k} \alpha_i} \]

where $\alpha_i \in [0,1]$ is the degree at which the antecedent of rule $R_i$ holds.

$\alpha_i$ is computed as in Mamdani Fuzzy Control.