10. Fuzzy Control based on Equality Relations

10.1

Problem:
Given a set of samples $(\zeta_1,\ldots,\zeta_n,\eta) \in X_1\times\ldots\times X_n\times Y$
What is the corresponding function $\varphi: X_1\times\ldots\times X_n\to Y$?

Idea:
Inputs almost equal to given samples should have almost the same output value. For this equality realions can be used.
The given samples can be interpreted as a set of fuzzy rules R_r: if ζ_1 is approximately x_1^r and … and ζ_n is approximately x_n^r then η is approximately y^r.

Example 10.2

Let $\varphi_0: X_1\times\ldots\times X_n\to Y$ be a partial defined function and $E_1,\ldots, E_n$, F equality functions on $X_1,\ldots,X_n$.
Instead of φ the extensional hull of the graph of φ is computed based on an equality relation E.

$$E((x_1,\ldots,x_n,y),(x_1',\ldots,x_n',y'))=\min\{E_1(x_1, x_1'),\ldots, E_n(x_n, x_n'), F(y,y')\}$$

$$\mu_{\varphi_0}(x_1',\ldots,x_n',y') = \max_{r\in\{1,\ldots,k\}}\left\{\min\left\{\begin{matrix}E_1\left(x_1',x_1^r\right),\ldots,\\ E_n\left(x_n',x_n^r\right),F\left(y',y^r\right)\end{matrix}\right\}\right\}$$

Theorem 10.3

Let $\mu^{\text{output}}_{x_1,\ldots,x_n}$ a fuzzy set, that is computed by a Mamdani fuzzy controller for a datum $(x_1,\ldots,x_n)$ with the following knowledge base:

- The partition of a set X_1 is given by the singletons $\mu^r_{x_i}$ defined by the given data $x_1^r,\ldots,x_n^r$.
- A fuzzy set μ_{x_0} is associated with the linguistic term approximately x_0.
- The rule basis has the rules R_r, $r=1,\ldots,k$.

Then (based on the notion of 10.2) $\mu^{(x_1,\ldots,x_n)}_{\varphi_0} = \mu^{\text{output}}_{x_1,\ldots,x_n}$ holds.

That is: **a fuzzy controller based on similarity relations can be interpreted as a Mamdani fuzzy controller.**

Interpolation Philosophy

Input $(d,\dot{d})$ Output $f(d,\dot{d})$

if d and d_i are similar and $\dot{d}$ and $\dot{d}_j$ are similar

then the output of $f(d,\dot{d})$ should be similar to l_{ij}

Result

For input $(d,\dot{d})$, the output should be

$$\mu_{d,\dot{d}}(l) = \sup_{i,j}\{\min(E_D(d,d_i), E_{\dot{D}}(\dot{d},\dot{d}_j),E_L(l,l_{ij}))\}$$

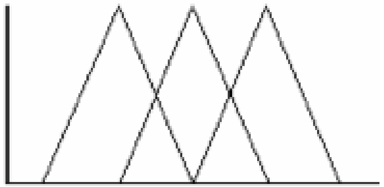
where $f(d,\dot{d})$ has to be obtained by defuzzification of $\mu_{d,\dot{d}}$

Observation: Same result as with Mamdani Fuzzy Control,

Fuzzy Mandani Control is “interpolation in vague environment“.

Reinterpretation of Mamdani Control

- Fuzzy partition on X



↔ Similarity relation on X

$E: X \times X \rightarrow [0, 1]$

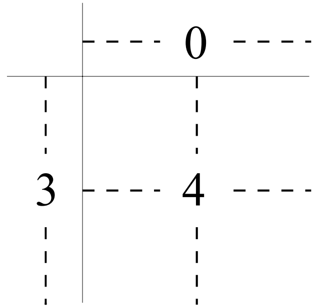
$E(x,x)=1$, $E(x,y)=E(x,y)$

$E(x,y)+E(y,x)-1 \leq E(x,z)$

- Fuzzy rule base

  if d ≈ 0 and d ≈ 3 then l ≈ 4

↔ Table



- Calculation of output

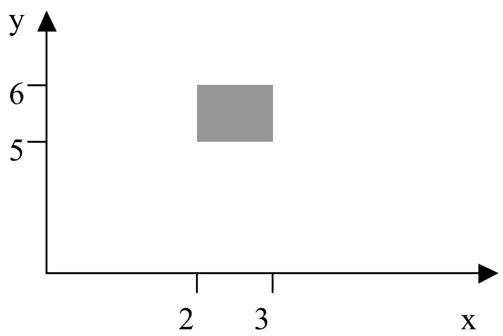
↔ Interpolation in similarity environment

⇨ Control function ≙ “typical“ input/output-tupel + similarity relation

Example 10.4

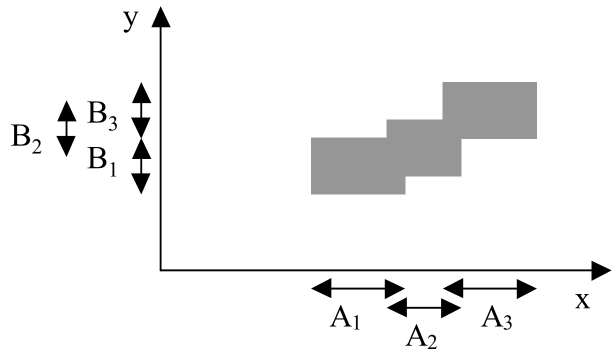
a) Imprecise rule:

if x ∈ [2,3] the y ∈ [5,6]

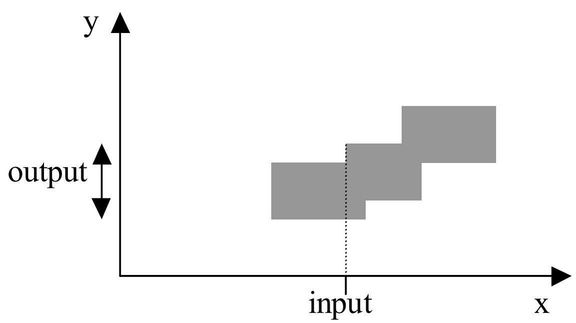


b) Set of imprecise rules:

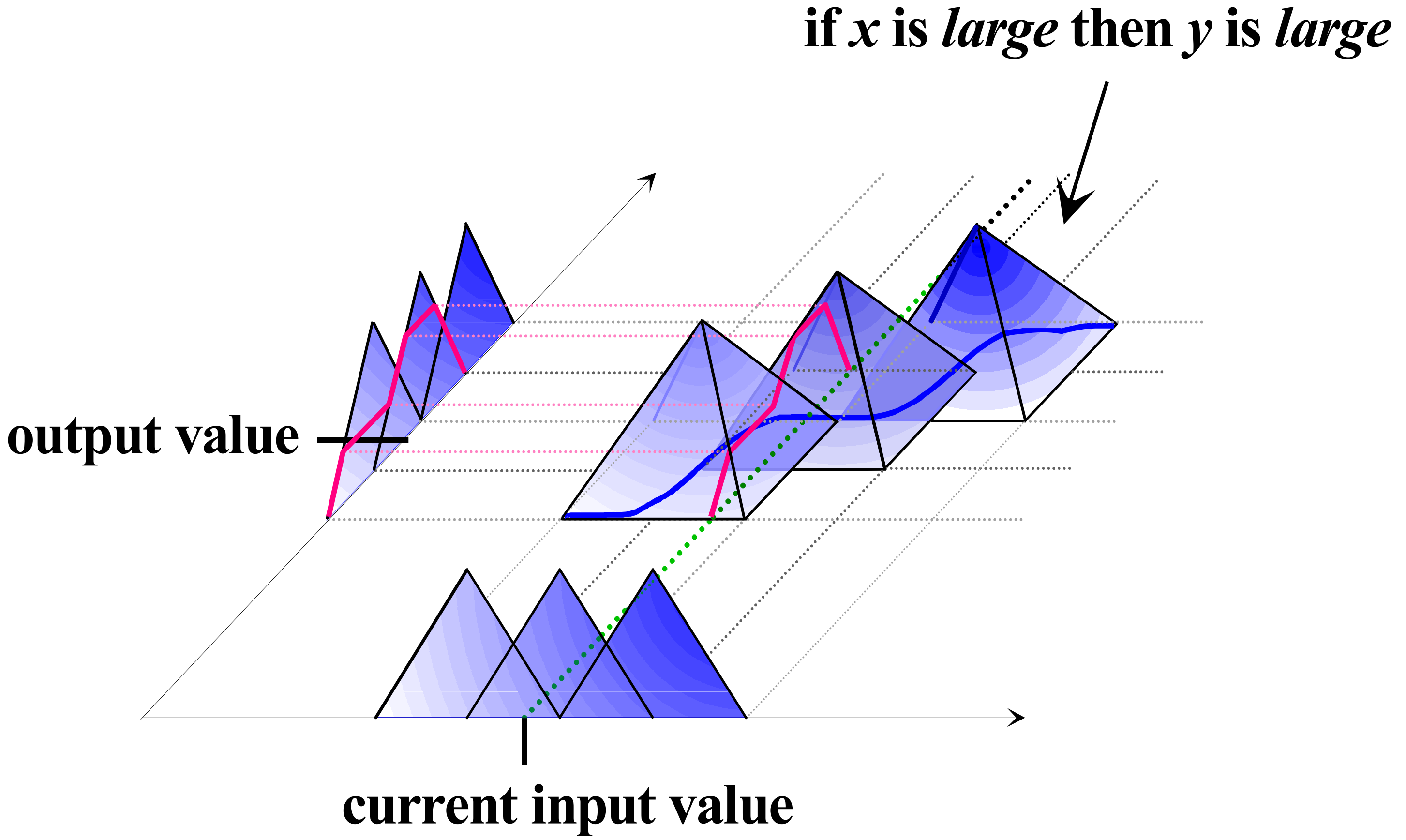
if A_i then B_i, i=1,2,3



c) Conclusion

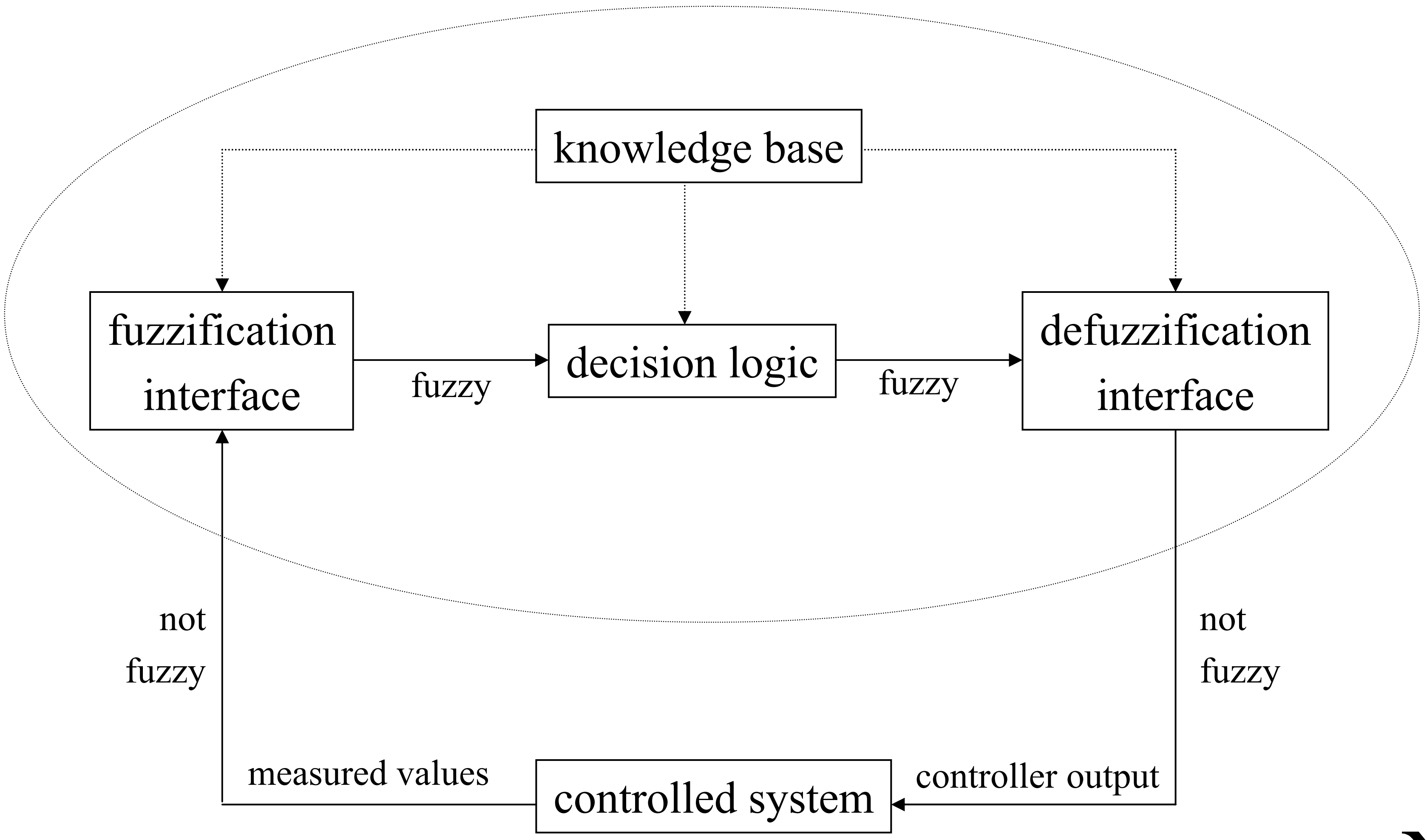


Example 10.5



if *x* is *large* then *y* is *large*
output value
current input value

Example 10.6. Fuzzy Controller



Example 10.7 Automatic Gear Box

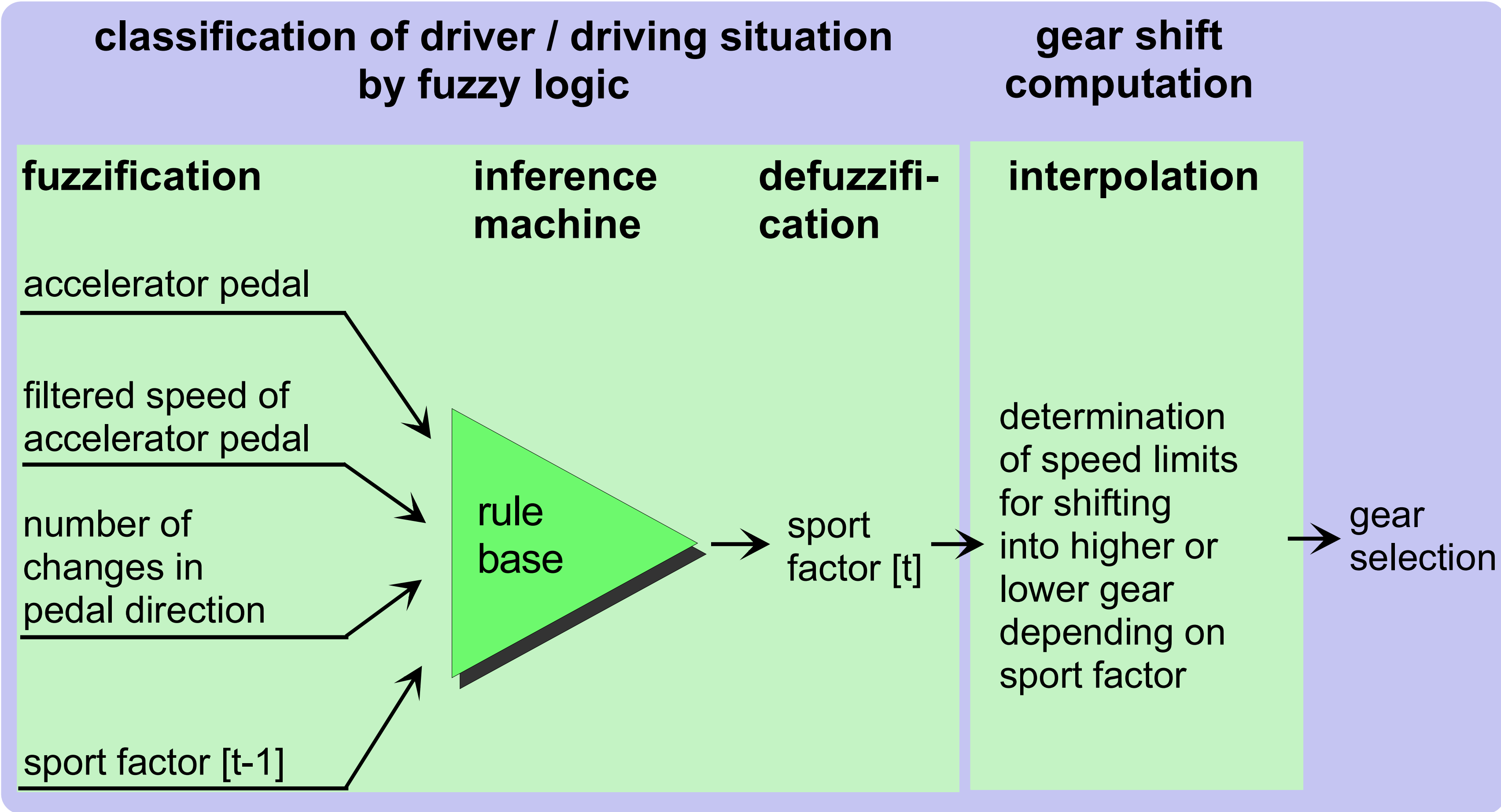
VW-gear box with two modes (ECO - SPORT) in series line until 1994

Research issue since 1991: - individual adaptation of set points
- no additional sensoric

Idea: car “watches“ driver and classifies driver:
calm, normal, sportive (assign sport factor (0,1))
nervous (calm down driver)

Test car: - different drivers, classification by expert (passenger)
- simultaneous measurement:
speed, position of accelerator pedal,
speed of accelerator pedal, kick down,
steering wheel angle, ... (14 attributes)

Continuously Adapting Gear Shift Schedule in VW New Beetle

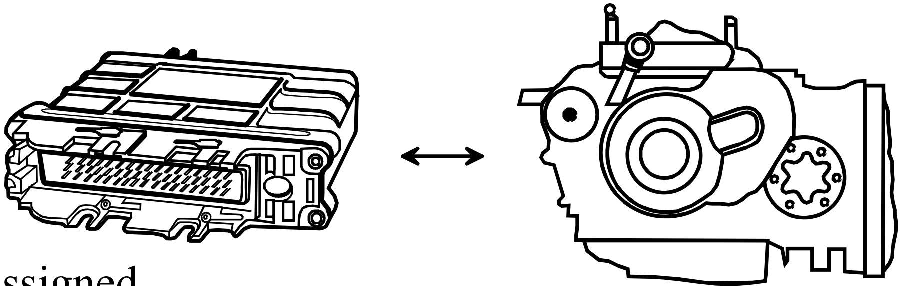


Continuously Adapting Gear Shift Schedule: Technical Details

- Mamdani controller with 7 rules
- Optimized program
  - 24 Byte RAM
  - 702 Byte ROM

  } on Digimat
- Runtime 80 ms
  12 times per second a new sport factor is assigned
- How to find suitable rules?

AG4



Resources

1. Kruse, Gebhardt, Klawonn, Foundations of Fuzzy systems, Wiley, 1996

2. Schröder, Petersen, Klawonn, Kruse, Two Paradigms of Automotive Fuzzy Logic Applications, in M. Jamshidi et al, Applications of Fuzzy Logic, Prentice Hall 1997

3. Michels, Klawonn, Kruse, Nürnberger Fuzzy Control, Springer, 2002