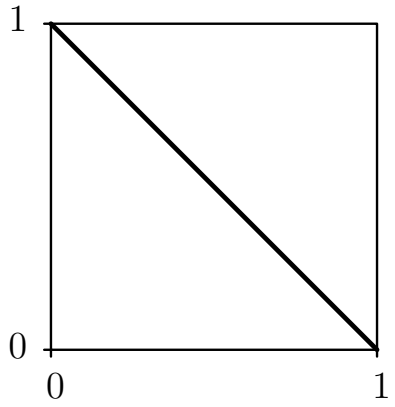
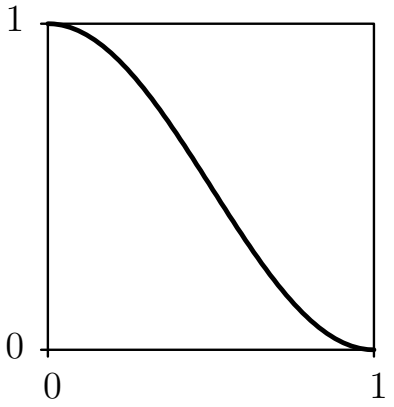
Fuzzy Negations

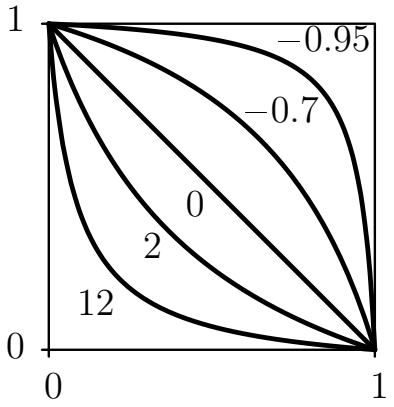
$$
\begin{array}{lllll}
\text{standard negation:} & \sim a & = & 1 - a \\
\text{threshold negation:} & \sim(a;\theta) & = & \begin{cases} 1, & \text{if } x \leq \theta, \\ 0, & \text{otherwise.} \end{cases} \\
\text{cosine negation:} & \sim a & = & \frac{1}{2}(1 + \cos \pi a) \\
\text{Sugeno negation:} & \sim(a;\lambda) & = & \dfrac{1-a}{1+\lambda a} \\
\text{Yager negation:} & \sim(a;\lambda) & = & (1 - a^{\lambda})^{\frac{1}{\lambda}}
\end{array}
$$



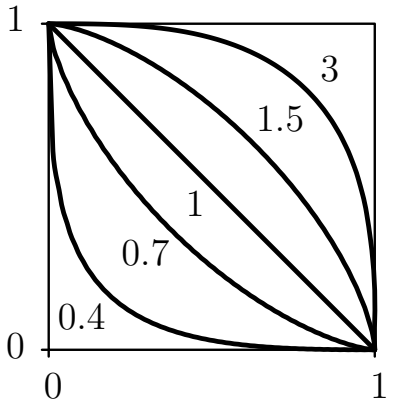
standard



cosine



Sugeno



Yager

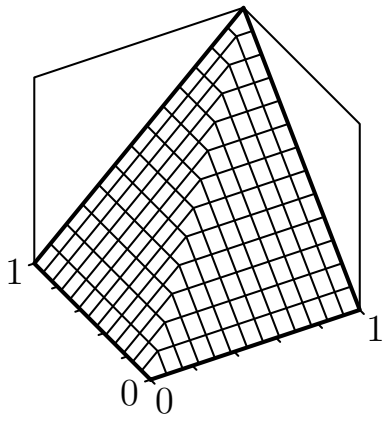
t-Norms / Fuzzy Conjunctions

standard conjunction: $\top_{\min}(a,b) = \min\{a,b\}$

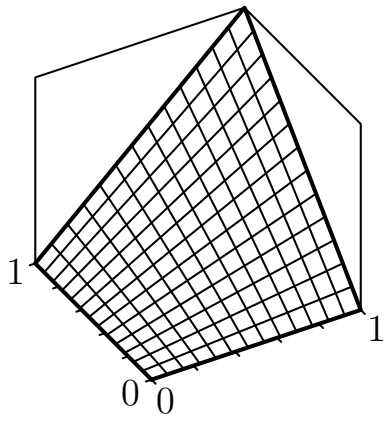
algebraic product: $\top_{\text{prod}}(a,b) = a \cdot b$

Łukasiewicz: $\top_{\text{Łuka}}(a,b) = \max\{0, a+b-1\}$

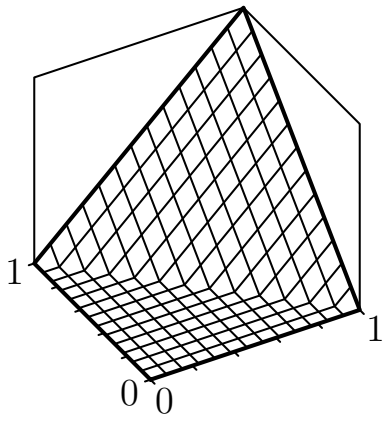
drastic product:
$$\top_{-1}(a,b) = \begin{cases} a, & \text{if } b = 1, \\ b, & \text{if } a = 1, \\ 0, & \text{otherwise.} \end{cases}$$



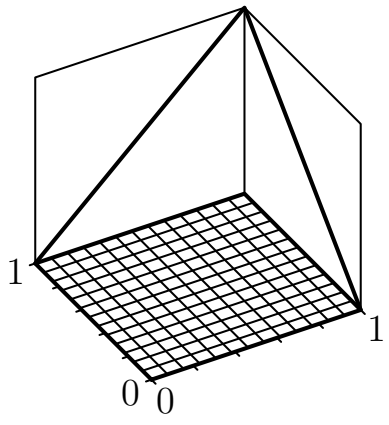
$\top_{\min}$



$\top_{\text{prod}}$



$\top_{\text{Łuka}}$



$\top_{-1}$

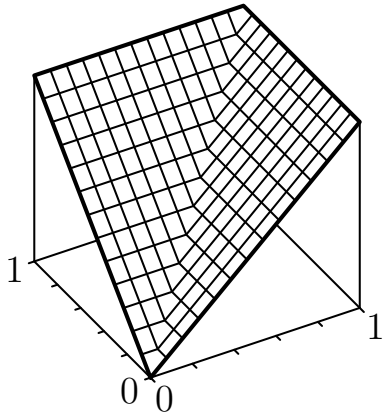
t-Conorms / Fuzzy Disjunctions

standard disjunction: $\perp_{\max}(a,b) = \max\{a,b\}$

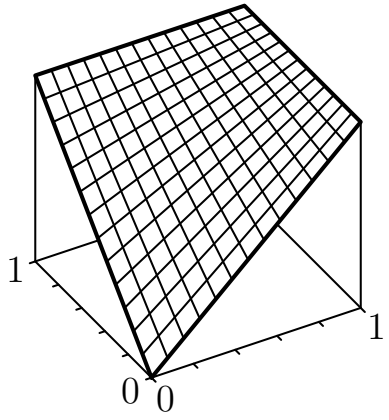
algebraic sum: $\perp_{\text{sum}}(a,b) = a + b - a \cdot b$

Łukasiewicz: $\perp_{\text{Łuka}}(a,b) = \min\{1, a+b\}$

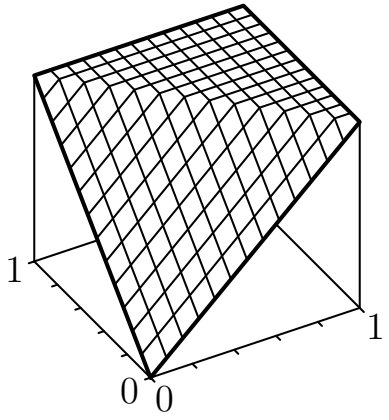
drastic sum:
$$\top_{-1}(a,b) = \begin{cases} a, & \text{if } b = 0, \\ b, & \text{if } a = 0, \\ 1, & \text{otherwise.} \end{cases}$$



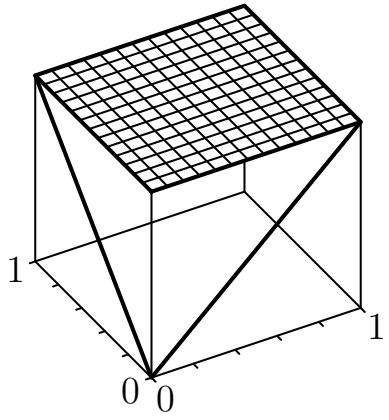
$\perp_{\max}$



$\perp_{\text{sum}}$



$\perp_{\text{Łuka}}$



$\perp_{-1}$