Exercise 9 \hspace{1cm} \textbf{Separation Criteria: d-Separation}

Consider the following directed graph:

\begin{center}
\begin{tikzpicture}
    \node (A) at (0,0) {A};
    \node (B) at (0,-2) {B};
    \node (C) at (1,-1) {C};
    \node (D) at (2,-1) {D};
    \node (E) at (3,-1) {E};
    \node (F) at (1,2) {F};
    \node (G) at (2.5,0) {G};
    \node (H) at (3.5,0) {H};
    \draw (A) -- (F);
    \draw (F) -- (G);
    \draw (G) -- (H);
    \draw (A) -- (C);
    \draw (A) -- (D);
    \draw (B) -- (C);
    \draw (D) -- (E);
\end{tikzpicture}
\end{center}

Which of the following propositions hold true in the graph??

\begin{align*}
\text{i)} & \quad F \perp \perp H \mid G \\
\text{ii)} & \quad C \perp \perp G \mid F \\
\text{iii)} & \quad F \perp \perp E \mid C \\
\text{iv)} & \quad A \perp \perp B \mid \emptyset \\
\text{v)} & \quad A \perp \perp B \mid D \\
\text{vi)} & \quad D \perp \perp F \mid \{C,G\} \\
\text{vii)} & \quad E \perp \perp F \mid \{A,B\} \\
\text{viii)} & \quad C \perp \perp E \mid \{D,F,H\}
\end{align*}

Exercise 10 \hspace{1cm} \textbf{Separation Criteria: u-Separation}

Consider the undirected graph that is obtained if all arrow heads from the directed graph in exercise 9 are dropped. Check again the propositions i)--viii) of exercise 9, now with the u-separation criterion! Which differences can be observed?

Exercise 11 \hspace{1cm} \textbf{Separation Criteria: d/u-Separation}

Remember the alternative way of checking for d-separation that was presented in the lecture (slides 56–58): \(X\) and \(Y\) are d-separated by \(Z\) if \(X\) and \(Y\) are u-separated by \(Z\) in the moralised minimal ancestral subgraph induced by \(X \cup Y \cup Z\). With this approach, verify again the results from exercise 9!