Markov Properties of Undirected Graphs

Let \((\cdot \perp \cdot \mid \cdot)\) be the ternary relation that represents the conditional independence statements that hold true in a probability distribution \(p\) over a common domain and set \(V\) of attributes. An undirected graph \(G = (V, E)\) satisfies the **pairwise Markov property** if and only if every pair of non-adjacent attributes in the graph are conditional independent in \(p\) given all other attributes, i.e.

\[
\forall A, B \in V, A \neq B : (A, B) \notin E \Rightarrow A \perp \perp B \mid V \setminus \{A, B\}.
\]

\(G\) has the **local Markov property** if and only if every attribute in \(p\) is conditionally independent of all others given its neighbors, i.e.

\[
\forall A \in V : A \perp \perp V \setminus \{A\} \mid \text{neighbors}(A),
\]

with \(\text{neighbors}(A) = \{B \in V \mid (A, B) \in E\}\).

\(G\) has the **global Markov property** if and only if from \(u\)-separation of two sets of attributes given a third one it follows that these two sets are conditionally independent in \(p\) given the third one, i.e.

\[
\forall X, Y, Z \subseteq V : \langle X \mid Z \mid Y \rangle_G \Rightarrow X \perp \perp Y \mid Z.
\]

**Exercise 29** Markov Properties of Undirected Graphs

Consider the following graph:

\[ \begin{array}{ccccc}
A & \rightarrow & B & \rightarrow & C \\
& & \rightarrow & \rightarrow & D \rightarrow E
\end{array} \]

Let \(\text{dom}(A) = \cdots = \text{dom}(E) = \{0, 1\}\). Assuming the probability distribution \(P(A = 0) = P(E = 0) = \frac{1}{2}\), \(A = B\) (i.e. \(P(B = 0 \mid A = 0) = 1\) and \(P(B = 1 \mid A = 1) = 1\)), \(D = E\) and \(C = B \cdot D\), show that the graph satisfies the pairwise and local but not the global Markov property.

**Exercise 30** Dempster-Shafer Theory

Specify for all following mass distributions over \(\Omega = \{1, 2, 3\}\) the respective belief and plausibility function (missing table entries denote 0).
Exercise 31  Dempster-Shafer Theory

Homicide was committed. The circle of suspects consists of three persons:

$$\Omega = \{\text{Antony, Beth, Charly}\}$$

We assume that exactly one of these persons has committed the homicide. Two witnesses provide us with the following evidence:

- $m_1(\{\text{Antony, Beth}\}) = 0.8$ und $m_1(\{\text{Charly}\}) = 0.2$

- $m_2(\{\text{Antony, Charly}\}) = 0.3$ und $m_2(\{\text{Beth}\}) = 0.7$

Calculate $m_1 \oplus m_2$ and $\text{Bel}_1 \oplus \text{Bel}_2$ for the arguments $\emptyset, \{\text{Antony}\}, \{\text{Beth}\}$ und $\{\text{Charly}\}$.