Exercise Sheet 4

Exercise 12 Marginal Distributions, (conditional) Independencies

Consider the contingency table with attributes M=Malaria, G=Flu, F=Fever and H=cough. The respective binary domains $\text{dom}(\mathsf{X}) = \{\mathsf{x}, \overline{\mathsf{x}}\}$ designate for $\mathsf{X} = \mathsf{M}, \mathsf{G}, \mathsf{F}, \mathsf{H}$ the meaning $\mathsf{x} \mathrel{\widehat{=}}$ "symptom/desease is present" and $\overline{\mathsf{x}} \mathrel{\widehat{=}}$ "symptom/desease is not present".

p_{MGFH}		$\mathsf{G}=\mathsf{g}$, $\mathsf{M}=\mathsf{m}$	$\mathsf{G}=\mathsf{g}$, $\mathsf{M}=\overline{\mathsf{m}}$	$\mathsf{G}=\overline{\mathsf{g}}$, $\mathsf{M}=\mathsf{m}$	$\mathsf{G}=\overline{\mathsf{g}}$, $\mathsf{M}=\overline{\mathsf{m}}$
$\mathsf{F}=\mathsf{f}$	$\mathsf{H}=\mathsf{h}$	144	1008	192	216
	$\mathsf{H}=\overline{\mathsf{h}}$	36	252	448	504
$\mathsf{F}=\overline{\mathsf{f}}$	$\mathsf{H}=\mathsf{h}$	16	432	48	1944
	$\mathsf{H}=\overline{\mathsf{h}}$	4	108	112	4536

a) Compute all four marginal distributions.

b) Compute $P(\mathsf{M}=\mathsf{m} \mid \mathsf{F}=\mathsf{f})$ and $P(\mathsf{G}=\mathsf{g} \mid \mathsf{F}=\mathsf{f})$, as well as $P(\mathsf{M}=\mathsf{m} \mid \mathsf{F}=\mathsf{f}, \mathsf{H}=\mathsf{h})$ and $P(\mathsf{G}=\mathsf{g} \mid \mathsf{F}=\mathsf{f}, \mathsf{H}=\mathsf{h})$.

c) Show that M and G are marginally independent, but conditionally dependent given F.

d) Show that F and H are marginally dependent, but conditionally independent given G.

Exercise 13 Decompositions of Relations

Consider the given relation over the three attributes A, B and C, whose domains contain three values each, i.e., $\text{dom}(A) = \{a_1, a_2, a_3\}$, $\text{dom}(B) = \{b_1, b_2, b_3\}$ and $\text{dom}(C) = \{c_1, c_2, c_3\}$. How can this relation be decomposed into projections onto subspaces? How can the original relation be reconstructed from these projections? Which (single) Tuples can be removed from the original relation without violating the decomposability? Why is it not possible to remove the other tuples? Which (single) tuples can be added to the original relation without violating the decomposability? Why can no other tuple be added?

A	B	C
a_1	b_1	c_1
a_1	b_1	c_2
a_2	b_1	c_2
a_2	b_1	c_3
a_2	b_3	c_2
a_2	b_3	c_3
a_3	b_1	c_2
a_3	b_2	c_2

Exercise 14 Conditional relational Independence

The relation from exercise 13 can be decomposed into projections because a conditional independence holds true. Which? How can this conditional independence be verified? (Hint: Remember that projections of relations can be represented by a maximum operation on the indicator function of the relation. Intersections of the cylindrical extensions can be modelled by a minimum operation. The indicator function of a relation returns value 1 for every tuple contained in the relation and returns value 0 for all others.)