Institute for Intelligent Cooperating Systems
Computational Intelligence
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Final Examination in "Fuzzy Systems"

Last Name, Surname:	Faculty:	Field of Studies:	Matricul. No.:
Type of Exam: ☐ Regular 1st/2nd Trial ☐ Ungraded Certificate ☐ Graded Certificate	Signature of Supervision:		No. Sheets:

Asgmt. 1	Asgmt. 2	Asgmt. 3	Asgmt. 4	Asgmt. 5	Total
/14	/12	/13	/8	/11	/58

Assignment 1 Fuzzy Arithmetic (4 + 3 + 3 + 4 = 14 points)

Consider the following triangular fuzzy numbers:

$$\mu_{l,m,r}(x) = \begin{cases} \frac{x-l}{m-l} & \text{if } l \le x \le m, \\ \frac{r-x}{r-m} & \text{if } m \le x \le r, \\ 0 & \text{otherwise} \end{cases}$$

where $l, m, r \in \mathbb{R}$ and $l < m < r$.

a) Compute the alpha-cut representation of $\mu_{l,m,r}$

Determine an explicit (horizontal) form of the following fuzzy sets (defined by the extension principle) using the alpha-cut representation. Which result(s) are triangular fuzzy sets?

b) $\mu_{3,4,5} + \mu_{1,2,3}$

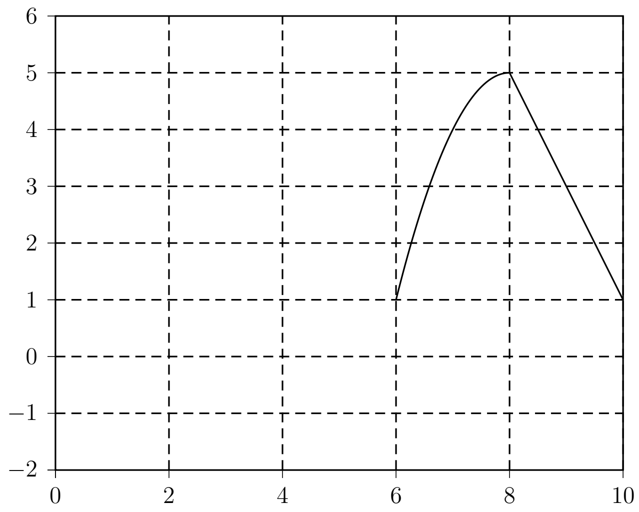
c) $2 \cdot \mu_{5,6,8} - \mu_{5,6,8}$

d) $\mu_{1,2,3} \cdot \mu_{5,6,8}$

Assignment 2 Takagi-Sugeno Controller (12 points)

From a Takagi-Sugeno Controller with one input and one output variable it is known that it uses triangular fuzzy sets $\mu_{k,k+2,k+4}$ for k $\in [0, 2, 4, 6]$ on the input interval $[0, 10]$. A part of the output function $f : [0, 10] \to [-2, 6]$ is shown on the right side. The following rules are known:

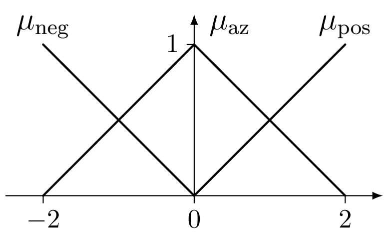
- if ξ is $\mu_{0,2,4}$, then $\eta_1(\xi) = 6 - \xi$
- if ξ is $\mu_{2,4,6}$, then $\eta_2(\xi) = -\frac{\xi}{2}$
- if ξ is $\mu_{4,6,8}$, then $\eta_3(\xi) = c$
- if ξ is $\mu_{6,8,10}$, then $\eta_4(\xi) =???$



Use these information to calculate the right hand side of the fourth rule. Compute the constant c and complement the drawing on the right side. Also calculate the general form of the output function.

Assignment 3 Mamdani-Assilian Controller (8 + 2 + 1 + 2 = 13 points)

Consider a Mamdani-Assilian controller with two inputs $\xi_1 \in X_1 = [-2, 2]$ and $\xi_2 \in X_2 = [-2, 2]$ and one output $\eta \in Y = [-2, 2]$. The utilized fuzzy partitions shall be the same for all three domains. They are shown below on the left ("az" means "approximately zero"). The rule base of the controller is shown on the right in tabular form.



ξ_2 \ ξ_1	neg	az	pos
neg	neg		az
az		neg	
pos	az		pos

a) Determine the fuzzy output of this controller for the two input tuples (-0.7,-0.4), (-0.5,1.2).

b) Determine crisp output values from the fuzzy outputs computed in part a) using the mean of maxima method.

c) Is the control function of b) continuous?

d) Describe two further defuzzification methods.

Assignment 4 Fuzzy Relational Equations (8 points)

Let $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$. Consider the fuzzy sets μ_1, μ_2, μ_3 on X and ν_1, ν_2, ν_3 on Y which are defined as shown below. Be $\mu_i \circ \varrho = \nu_i$ for all $i = 1, 2, 3$ a system of fuzzy relational equations.

$\mu_i(x)$	x_1	x_2	x_3
μ_1	0.9	0.6	0.3
μ_2	0.1	0.3	0.7
μ_3	0.5	0.8	0.2

$\nu_i(y)$	y_1	y_2
ν_1	0.3	0.6
ν_2	0.7	0.5
ν_3	0.2	0.7

What is the greatest solution of this system?

Assignment 5 Multiple Choice (4 + 4 + 3 = 11 points)

Please mark the correct answers of each question with a cross.
You gain one point per correctly marked answer and lose one point for each wrong answer. Skipped questions leave your score unchanged. For each sub-task you gain at least 0 points, so points achieved in other sub-tasks will always be kept.

a) Consider the function family consisting of two linear functions: $\sim_{a,b}\colon [0,1] \to [0,1]$ for $a, b \in]0,1[$, which go through the points (0,1), (a,b) and (1,0) as shown below. Given a and b, then

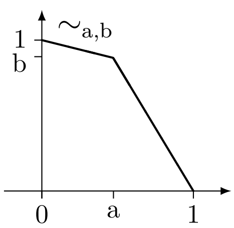
yes no

☐ ☐ $\sim_{a,b}$ is involutive.

☐ ☐ $\sim_{a,b}$ is strictly decreasing.

☐ ☐ $\sim_{a,b}$ forms a fuzzy negation.

☐ ☐ $\sim_{a,b}$ is continuous.



b) Consider Fuzzy C-Means clustering with four clusters, distance measure d and n data points. Be p_1 and p_2 data points, c_1 a cluster prototype with $d(p_1, c_1) < d(p_2, c_1)$ and $u_{p1,c1}, u_{p2,c1}$ the corresponding membership values.

yes no

☐ ☐ For each of the n data points, the sum of its fuzzy membership degrees to the four clusters equals 1.

☐ ☐ The inequality $u_{p1,c1} < u_{p2,c1}$ holds.

☐ ☐ The inequality $u_{p1,c1} > u_{p2,c1}$ holds.

☐ ☐ Higher values of the *fuzzifier* lead to softer clusters.

c) Fuzzy Logic

yes no

☐ ☐ The fuzzy logic proposed by Zadeh in 1965 forms a Boolean Algebra.

☐ ☐ Given a strict, involutive negation $\sim$ and a t-norm $\top$, then a so called dual t-conorm $\perp$ exists.

☐ ☐ Given a t-norm $\top$ and a t-conorm $\perp$, then a negation $\sim$ can be defined, such that $(\top,\perp,\sim)$ build a *De Morgan triplet*.