Fuzzy Systems
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Assignment Sheet 12

Assignment 44 Fuzzy Clustering

Consider the one-dimensional data set

$$1, 3, 4, 5, 8, 10, 11, 12.$$

We want to process this data set with fuzzy c-means clustering using $c = 2$ (two clusters) and the fuzzifier $m = 2$. Assume that the cluster centers are initialized to 1 and 5. Execute one step of alternating optimization as it is used for fuzzy clustering, *i.e.*

a) Compute the membership degrees of the data points for the initial cluster centers.

b) Compute new cluster centers from the membership degrees that have been obtained before.

Assignment 45 Fuzzifier m

Consider the objective function of fuzzy clustering with a fuzzifier $m \geq 1$, *i.e.*

$$J_f(X, U, C) = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^m d^2(\mathbf{c}_i, \mathbf{x}_j) \quad \text{subject to} \quad \forall j \in \{1, \ldots, n\} : \sum_{i=1}^{c} u_{ij} = 1.$$

Assume that the minimum of J_f is obtained $\forall i \in \{1, \ldots, c\} : \forall j \in \{1, \ldots, n\} : d(\mathbf{c}_i, \mathbf{x}_j) > 0$, *i.e.* the cluster centers do not coincide with any data points.

a) Show that if the fuzzifier $m = 1$ one obtains hard/crisp assignments of data points even if the membership degrees $u_{ij} \in [0, 1]$. Thus, show that the minimum of J_f is attained $\forall i \in \{1, \ldots, c\} : \forall j \in \{1, \ldots, n\} : u_{ij} \in \{0, 1\}$.

b) Show that if the fuzzifier $m > 1$ one cannot obtain hard/crisp assignments of data points even if the membership degrees $u_{ij} \in [0, 1]$. Thus, show that the minimum of J_f is attained $\forall i \in \{1, \ldots, c\} : \forall j \in \{1, \ldots, n\} : u_{ij} \in]0, 1[$.

Hint: You may find it easier to consider the special case $c = 2$ (two clusters) and to examine the term for a single data point $\mathbf{x}_j$.

Assignment 46 Fuzzy Clustering

Consider the objective function

$$J_f(X, U, C) = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij} d^2(\mathbf{c}_i, \mathbf{x}_j),$$

subject to

$$\forall j \in \{1, \ldots, n\} : \sum_{i=1}^{c} \sqrt{u_{ij}} = 1.$$

a) Derive the update formulas for the membership degrees and the cluster centers using the Euclidean distance.

b) How does the result differ from standard fuzzy clustering with a fuzzifier $m = 2$? (In particular, consider the cluster centers.)

Assignment 47 Noise Clustering

Show that the noise clustering (NC) algorithm and the possibilistic c-means (PCM) algorithm are identical in the case of a single cluster $c = 1$, with δ^2 corresponding to $\eta = \eta_1$.