Assignment Sheet 12

Assignment 44  Fuzzy Clustering

Consider the one-dimensional data set
\[ 1, 3, 4, 5, 8, 10, 11, 12. \]
We want to process this data set with fuzzy \( c \)-means clustering using \( c = 2 \) (two clusters) and the fuzzifier \( m = 2 \). Assume that the cluster centers are initialized to 1 and 5. Execute one step of alternating optimization as it is used for fuzzy clustering, i.e.

a) Compute the membership degrees of the data points for the initial cluster centers.

b) Compute new cluster centers from the membership degrees that have been obtained before.

Assignment 45  Fuzzifier \( m \)

Consider the objective function of fuzzy clustering with a fuzzifier \( m \geq 1 \), i.e.
\[
J_f(X, U, C) = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^m d^2(c_i, x_j) \quad \text{subject to} \quad \forall j \in \{1, \ldots, n\} : \sum_{i=1}^{c} u_{ij} = 1.
\]
Assume that the minimum of \( J_f \) is obtained \( \forall i \in \{1, \ldots, c\} : \forall j \in \{1, \ldots, n\} : d(c_i, x_j) > 0 \), i.e. the cluster centers do not coincide with any data points.

a) Show that if the fuzzifier \( m = 1 \) one obtains hard/crisp assignments of data points even if the membership degrees \( u_{ij} \in [0, 1] \). Thus, show that the minimum of \( J_f \) is attained \( \forall i \in \{1, \ldots, c\} : \forall j \in \{1, \ldots, n\} : u_{ij} \in \{0, 1\} \).

b) Show that if the fuzzifier \( m > 1 \) one cannot obtain hard/crisp assignments of data points even if the membership degrees \( u_{ij} \in [0, 1] \). Thus, show that the minimum of \( J_f \) is attained \( \forall i \in \{1, \ldots, c\} : \forall j \in \{1, \ldots, n\} : u_{ij} \in ]0, 1[ \).

Hint: You may find it easier to consider the special case \( c = 2 \) (two clusters) and to examine the term for a single data point \( x_j \).

Assignment 46  Fuzzy Clustering

Consider the objective function
\[
J_f(X, U, C) = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij} d^2(c_i, x_j),
\]
subject to
\[
\forall j \in \{1, \ldots, n\} : \sum_{i=1}^{c} \sqrt{u_{ij}} = 1.
\]
a) Derive the update formulas for the membership degrees and the cluster centers using the Euclidean distance.

b) How does the result differ from standard fuzzy clustering with a fuzzifier $m = 2$? (In particular, consider the cluster centers.)

Assignment 47 Noise Clustering

Show that the noise clustering (NC) algorithm and the possibilistic $c$-means (PCM) algorithm are identical in the case of a single cluster $c = 1$, with $\delta^2$ corresponding to $\eta = \eta_1$. 