Assignment Sheet 10

Assignment 37    Quantifiers

Assuming the conditions of Assignment 27, what fuzzy truth values do you get for the proposition “There exists a small single-digit prime number in the decimal number system”? Which problem arises thereby?

Hint: Directly use the dual $t$-conorms $\perp_{\max}(a, b) = \max\{a, b\}$ and $\perp_{\sum}(a, b) = a + b - a \cdot b$.

Assignment 38    Programming

Implement necessary data structures and algorithms for a Mamdani-Assilian controller that shall operate on two input variables $\xi_1$ and $\xi_2$. In detail, implement the following steps:

a) Enable the user to enter the ranges for $\xi_1$, $\xi_2$ and for $\eta$.

b) Implement an interface s.t. the user can specify linguistic terms for a given variable.

c) Implement a routine that partitions a variable by letting the user define membership functions for all linguistic terms of the chosen variable.

d) In order to input a fuzzy rule base, implement a method that enables the user to specify a linguistic term as output for all combinations of linguistic terms of $\xi_1$ and $\xi_2$. This can be done effectively by representing the rule base as a two-dimensional matrix.

e) Implement a method that creates a fuzzy set as control output for a given ($\xi_1, \xi_2$).

f) Implement the three defuzzification methods discussed in the lecture.

g) Create a method that randomly generates $l$ input tuples ($\xi_1, \xi_2$) and automatically predicts their outputs $\eta$ given your fuzzy controller. This three-dimensional data set shall be stored as comma separated values (CSV) file.

h) Put all parts together into a main method s.t. the user can create a fuzzy controller.

i) Verify if your code produces the same results for parts a) and b) of Assignment 34.

Assignment 39    Takagi-Sugeno Controller

Construct a Takagi-Sugeno controller with two inputs and one output that computes the following (partially defined) function (cf. Assignment 35):

\[
\begin{align*}
(1, 0) & \mapsto 2, \quad (1, 3) \mapsto 4, \\
(0, 2) & \mapsto 2, \quad (2, 2) \mapsto 4, \\
(2, 0) & \mapsto 2.
\end{align*}
\]

Determine the output of your controller for the inputs $(1, 1)$ and $(1.5, 1.5)$. 
Assignment 40  Takagi-Sugeno Controller

Consider the following definition of triangular fuzzy numbers

\[ \mu_{l,m,r} = \begin{cases} \frac{x-l}{m-l} & \text{if } l \leq x \leq m, \\ \frac{r-x}{r-m} & \text{if } m \leq x \leq r, \\ 0 & \text{otherwise} \end{cases} \]

whereas \( l, m, r \in \mathbb{R} \) and \( l < m < r \). Now, let a Takagi-Sugeno controller with the rule base be given as follows

- \( R_1 : \text{if } x \text{ is } \mu_1 \text{ then } y = 2, \)
- \( R_2 : \text{if } x \text{ is } \mu_2 \text{ then } y = x, \)
- \( R_3 : \text{if } x \text{ is } \mu_3 \text{ then } y = 3 - x^2, \)

whereas \( x \in X = [0, 8] \) and \( X \) is partitioned by \( \mu_1 = \mu_{0,2,4}, \mu_2 = \mu_{2,4,6}, \mu_3 = \mu_{4,6,8} \).

a) Compute the output of the controller by using the weighted sum

\[ f(x) = \frac{\sum_{r=1}^{3} \mu_{R_r}(x) \cdot f_{R_r}(x)}{\sum_{r=1}^{3} \mu_{R_r}(x)}, \]

whereas \( \mu_{R_r}(x) \) is the degree of fulfillment that the rule \( R_r \) “fires”, and \( f_{R_r} \) is the output of the rule \( R_r \).

b) Draw the output into a diagram.