OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG | INF | FACULTY OF COMPUTER SCIENCE

Department for Knowledge and Language Engineering
Computational Intelligence
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Magdeburg, February 7, 2013

Final Examination in "Fuzzy Systems"

last name, surname:		faculty:	field of studies:	matricul. no.:
type of exam:	☐ regular 1st/2nd trial ☐ ungraded certificate ☐ graded certificate	signature of supervision:		no. sheets:

Asgmt. 1	Asgmt. 2	Asgmt. 3	Asgmt. 4	Asgmt. 5	Asgmt. 6	Total
/5	/5	/10	/10	/10	/10+1	/50+1

Assignment 1 Duality of *t*-norm and *t*-conorm (5 points, ca. 10 minutes)

Show that the following pairs of a t-norm $\top$ and a t-conorm $\perp$ are dual with respect to the standard fuzzy negation $\sim a = 1 - a$:

a) $\top_{\min}(a, b) = \min\{a, b\}$ and $\perp_{\max}(a, b) = \max\{a, b\}$

b) $\top_{\text{prod}}(a, b) = a \cdot b$ and $\perp_{\text{sum}}(a, b) = a + b - ab$

Reminder: A t-norm $\top$ and a t-conorm $\perp$ are called dual with respect to a negation $\sim$ if and only if the fuzzy analogs of De Morgan's laws are satisfied.

Assignment 2 Idempotency (5 points, ca. 10 minutes)

Prove the following theorem.

Theorem: *The fuzzy conjunction* $\top_{\min}(a, b) = \min\{a, b\}$ *is the only idempotent t-norm.*

Reminder: A t-norm $\top$ is called idempotent if and only if $\forall a \in [0, 1] : \top(a, a) = a$.

Assignment 3 α-cut (10 points, ca. 20 minutes)

Determine the set representation of

$$\mu : \mathbb{R} \to [0, 1], \qquad \mu(x) = \begin{cases} \frac{1}{2}x + \frac{1}{2} & \text{if } -1 \leq x \leq 1 \\ 2 - x & \text{if } 1 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

Also, calculate and draw a sketch of the reciprocal value of μ.

Assignment 4 Extension Principle (10 points, ca. 20 minutes)

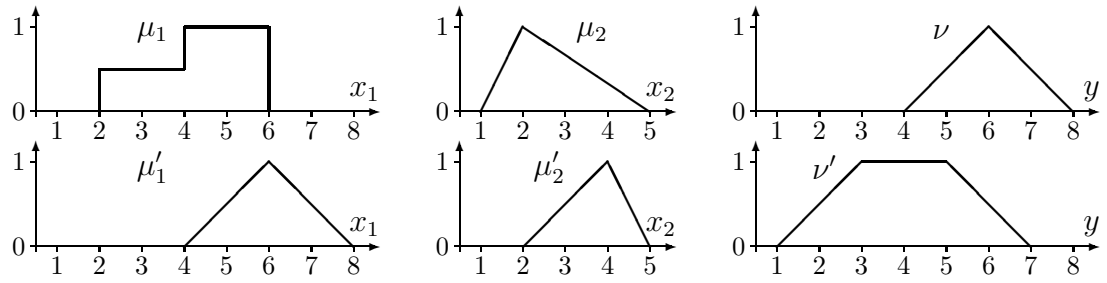
Let $\phi : \mathbb{N} \times \mathbb{N} \to \mathbb{N}, \quad \phi(a, b) = \max\{a, b\}$ and $\mu_1 : \mathbb{N} \to [0, 1], \quad \mu_2 : \mathbb{N} \to [0, 1]$ with

$$\begin{array}{lllll} \mu_1(1) = 1, & \mu_1(2) = 0.5, & \mu_1(3) = 0.2, & \mu_1(4) = 0.1, & \mu_1(n) = 0, \quad \text{for } n \geq 5, \\ \mu_2(1) = 0.1, & \mu_2(2) = 0.4, & \mu_2(3) = 0.9, & \mu_2(n) = 0, & \quad \text{for } n \geq 4. \end{array}$$

Determine $\hat{\phi}(\mu_1, \mu_2)$ according to the extension principle.

Assignment 5 Mamdani-Assilian Controller (10 points, ca. 20 minutes)

Let the following fuzzy sets and rules be given:



$$\begin{array}{lllllll} R_1: & \text{if} & x_1 \text{ is } \mu_1 & \text{and} & x_2 \text{ is } \mu_2 & \text{then} & y \text{ is } \nu \\ R_2: & \text{if} & x_1 \text{ is } \mu_1' & \text{and} & x_2 \text{ is } \mu_2' & \text{then} & y \text{ is } \nu' \end{array}$$

a) Based on these fuzzy sets and the rules, which output μ_{output} does a Mamdani-Assilian controller return for the input tuple $(5, 2.5)$?

b) Which crisp output values are (approximately) obtained by defuzzification of the output set with both mean of maxima method and center of gravity method?

Assignment 6 Automatic Distance Control (10 + 1 points, ca. 20 minutes)

Design a Mamdani-Assilian controller for the operation of an unmanned vehicle in a moving convoy. The goal is to keep a safety distance of half of the speedometer reading in meters to vehicle driving ahead of you. This can be performed by controlling both throttle (accelerator pedal position) and deceleration (brake pedal position). The vehicle is equipped with sensors which determine the distance to the vehicle driving ahead and the current longitudinal velocity. For simplicity we assume that the convoy moves just straight ahead without any curves.

a) Define linguistic variables to describe the necessary variables by choosing appropriate fuzzy sets. Draw every linguistic variable in a draft.

b) Suggest operations to compute the rule activations, to combine outputs of simultaneously active rules, and to defuzzify the final output fuzzy set.

c) Specify a set of rules that is suitable to control the vehicle.

d) *Extra:* Which conditions should be satisfied regarding the control variables?