# **Decision Theory**

Descriptive Decision Theorie tries to simulate human behavior in finding the right or best decision for a given problem

Example:

- Company can chose one of two places for a new store
- $\circ\,$  Option 1: 125.000 EUR profit per year
- Option 2: 150.000 EUR profit per year

Company should take Option 2, because it maximized the profit.

Often, there are multiple target values, which should be optimal

Example (additional Information):

- $\circ\,$  Option 1: 2.000.000 EUR sales per year
- $\circ\,$  Option 2: 1.800.000 EUR sales per year

There is a conflict to handle

In real world not every thing is known, so there are uncertainties in the model

Example:

- There are plans for restructure the local traffic, which changes the predicted profit
- Option 1: 125.000 EUR profit per year
- $\circ\,$  Option 2: 80.000 EUR profit per year

With modification Option 1 is the better one and without modification Option 2 is the better one

To model these variations in the environment we use so called Decision Tables

	$z_1$ (no modification)	$z_2$ (restructure)
$a_1$ (Option 1) $a_2$ (Option 2)	$125.000 = e_{11} \\ 150.000 = e_{21}$	$125.000 = e_{12} \\ 80.000 = e_{22}$

In many cases probabilities could be assigned to each option

**Objective Probabilities** based on mathematic or statistic background

Subjective Probabilities based on intuition or estimations

Example:

 $\circ\,$  The management estimates the probability for the restructure to  $30\%\,$ 

The decision can be chosen by expectation value

	$z_1$ (no modification) $p_1 = 0.7$	$z_2$ (restructure) $p_2 = 0.3$	Expectation Value
$a_1$ (Option 1) $a_2$ (Option 2)	$125.000 = e_{11} \\ 150.000 = e_{21}$	$125.000 = e_{12} \\ 80.000 = e_{22}$	$125.000 \\ 129.000$

Option 2 has the higher expectation value and should be used

## Domination

An alternative  $a_1$  dominates  $a_2$  if the value of  $a_1$  is always greater of (or equal to) the value of  $a_2$ 

$$\forall_j e_{1j} \ge e_{2j}$$
 '

Example:

	$z_1$	$z_2$
$a_1 a_2$	$150.000 = e_{11}$ $125.000 = e_{21}$	$90.000 = e_{12}$ $80.000 = e_{22}$

Alternative  $a_2$  could be dropped

## **Domination - Example 2**

Some more alternatives:

	$z_1$	$z_2$	$z_3$	$z_4$	$z_5$	
$a_1$	0	20	10	60	25	dominated by $a_3$
$a_2$	-20	80	10	10	60	
$a_3$	20	60	20	60	50	
$a_4$	55	40	60	10	40	
$a_5$	50	10	30	5	20	dominated by $a_4$

- $a_3$  dominated  $a_1$
- $a_4$  dominated  $a_5$ Alternatives  $a_1$  and  $a_5$  could be dropped

# **Probability Domination**

	$z_1 p_1 = 0.3$	$z_2 \\ p_2 = 0.2$	$z_3 p_1 = 0.4$	$p_2 = 0.1$
$a_1$	20	40	10	50
$a_2$	60	30	50	20

Probability Domination means that the cumulated probability for the payout for is always higher

### Algorithm:

- Order payout by value in a decreasing order
- Cumulate probabilities

## Example:

$$\circ a_1$$
: 50(0.1) 40(0.2) 20(0.3) 10(0.4)  
 $\circ a_2$ : 60(0.3) 50(0.4) 30(0.2) 20(0.1)

# **Probability Domination**



 $a_2$  dominates  $a_1$ .

## Multi Criteria Decisions

Optimization for multiple Targets

Complementary TargetsSelling left foot shoes / Selling right foot shoesOne could be avoided

Independent TargetsCould be optimized separately

Competitive Targets

- $\circ\,$  Increase profit and sales
- Decrease environment pollution

	Price	Sales $e_1$	Profit $e_2$	Environment Pollution $e_3$
$a_1$	15	800	7000	-4
$a_2$	20	600	7000	-2
$a_3$	25	400	6000	0
$a_4$	30	200	4000	0

### **Efficient Alternatives**

- Only focus on alternatives which are not dominated by others
- Example: Drop  $a_4$

## Finding a decision

- If multiple alternatives are effective we need an algorithm to choose the preferred one
- Simplest algorithm: Chose one target (most important, alphabetical) and optimize for this value

# Multi Criteria Decisions - Utility Function

Goal find a function  $U(e_1, e_2, \ldots, e_n)$  as a combination of all targets, which could be optimized

#### Linear combination

• Simplest variant: Linear combination of all targets

• 
$$U(e_1, e_2, \dots, e_i) = \sum_{i=1}^n \omega_i \cdot e_i$$

### Example

• 
$$\omega_1 = 10$$
,  $\omega_2 = 1$ ,  $\omega_3 = 500$ 

	Price	Sales $e_1$	Profit $e_2$	Environment Pollution $e_3$	$U(e_1, e_2, e_3)$
$a_1$	15	800	7000	-4	13000
$a_2$	20	600	7000	-2	12000
$a_3$	25	400	6000	0	10000

## **Decision under Uncertainty**

	$z_1$	$z_2$	$z_3$	$z_4$
$a_1$	60	30	50	60
$a_2$	10	10	10	140
$a_3$	-30	100	120	130

Think about, how you would decide!

### **Decision Rules**

- Maximin Rule
- Maximax Rule
- Hurwicz Rule
- Savage-Niehans Rule
- $\circ~$  Laplace Rule

## Maximin - Rule

	$z_1$	$z_2$	$z_3$	$z_4$	Minimum
$a_1$	60	30	50	60	30
$a_2$	10	10	10	140	10
$a_3$	-30	100	120	130	-30

Chose the one with the highest minimum

Contra: To pessimistic, only focus on one column

### Example

	$z_1$	$z_2$	$z_3$	$z_4$	Minimum
$a_1$	1,000,000	1,000,000	0.99	1,000,000	0.99
$a_2$	1	1	1	1	1

## Maximax - Rule

	$z_1$	$z_2$	$z_3$	$z_4$	Maximum
$a_1$	60	30	50	60	60
$a_2$	10	10	10	140	140
$a_3$	-30	100	120	130	130

### Chose the one with the highest maximum

### Contra: To optimistic, only focus on one column

### Example

	$z_1$	$z_2$	$z_3$	$z_4$	Maximum
$a_1$	1,000,000	$1,\!000,\!000$	$1,\!000,\!000$	$1,\!000,\!000$	$1,\!000,\!000$
$a_2$	$1,\!000,\!001$	1	1	1	$1,\!000,\!001$

	$z_1$	$z_2$	$z_3$	$z_4$	Max	Min	$\Phi(a_i)$
$a_1$	60	30	50	60	60	30	$0.4 \cdot 60 + 0.6 \cdot 30 = 42$
$a_2$	10	10	10	140	140	10	$0.4 \cdot 140 + 0.6 \cdot 10 = 62$
$a_3$	-30	100	120	130	130	-30	$0.4 \cdot 130 + 0.6 \cdot (-30) = 34$

Combination of Maximin and Maximax - Rule  $\Phi(a) = \lambda \cdot \max(e_i) + (1 - \lambda) \cdot \min(e_i)$   $\lambda \text{ represents readiness to assume risk}$  **Contra**: Only focus on two column Example (min(a<sub>1</sub>) < min(a<sub>2</sub>), max(a<sub>1</sub>) < max(a<sub>2</sub>)  $\Rightarrow$  chose a<sub>2</sub>)

	$z_1$	$z_2$	$z_3$	$z_4$	Max	Min
$a_1$	1,000,000	1,000,000	1,000,000	0.99	$1,\!000,\!000$	0.99
$a_2$	$1,\!000,\!001$	1	1	1	$1,\!000,\!001$	1

## Savage-Niehans - Rule

	$z_1$	$z_2$	$z_3$	$z_4$
$a_1$	60	30	50	60
$a_2$	10	10	10	140
$a_3$	-30	100	120	130

Rule of minimal regret

### Algorithm:

- Find the maximal value for every column
- Subtract value from maximal value
- Use alternative with the lowest regret

Regret Table:

	$z_1$	$z_2$	$z_3$	$z_4$	Max
$a_1$	60 - 60 = 0	70	70	80	80
$a_2$	60 - 10 = 50	90	110	0	110
$a_3$	60 - (-30) = 90	0	0	10	90

## Savage-Niehans - Rule II

	$z_1$	$z_2$	$z_3$	$z_4$
$a_1$ $a_2$	$1,000 \\ 1.001$	1,000,000	1,000,000	1,000,000

Another example

### we chose $a_1$

### Regret Table:

	$z_1$	$z_2$	$z_3$	$z_4$	Max
$a_1$	1	0	0	0	1
$a_2$	0	$1,\!000,\!000$	$1,\!000,\!000$	$1,\!000,\!000$	$1,\!000,\!000$

## Savage-Niehans - Rule III

	$z_1$	$z_2$	$z_3$	$z_4$
$a_1$	1,000	1,000,000	$1,\!000,\!000$	$1,\!000,\!000$
$a_2$	$1,\!001$	0	0	0
$a_3$	2,000,000	-1,000,000	-1,000,000	-1,000,000

Same example, but we add alternative  $a_3$ 

Now we chose  $a_2$ 

Regret Table:

	$z_1$	$z_2$	$z_3$	$z_4$	Max
$a_1$	1,999,000	0	0	0	1,999,000
$a_2$	1,998,999	$1,\!000,\!000$	$1,\!000,\!000$	$1,\!000,\!000$	$1,\!998,\!999$
$a_2$	0	$2,\!000,\!000$	$2,\!000,\!000$	$2,\!000,\!000$	$2,\!000,\!000$

## Savage-Niehans - Rule IV

### What this means in real life:

- Student think about swimming  $a_1$  and running  $a_2$
- The fun factor is depending on the weather  $z_1 \dots z_4$
- Student decides to go swimming
- He talk to a friend and presents his plans for the evening
- The friend mentioned to go for a BBQ  $a_3$
- With the option for BBQ the student decides to go running

## Laplace - Rule

	$z_1$	$z_2$	$z_3$	$z_4$	Mean
$a_1$	60	30	50	60	50
$a_2$	10	10	10	140	42.5
$a_3$	-30	100	120	130	80

Chose the one with the highest mean value

### Contra:

- Not every condition has the same probability
- Duplication of one condition could change the result

Most people would also chose  $a_3$  in this example

The following axioms should be fulfilled by the rules

## Addition to a column

The decision should not be changed, if a fixed value is added to a column

## Additional rows

The preference relation between two alternatives should not be changed, if a new row is added

### Domination

If  $a_1$  dominates  $a_2$ ,  $a_2$  could not be optimal

### Join of equal columns

The preference relation between to alternatives should not change, if two columns with the same outcomes are joined to a common column

Rule	Example Result	Addition to a row	Additional Rows	Domination	Join of equal Rows
Maximin Maximax Hurwicz	$egin{array}{c} a_1\ a_2\ a_2\ a_2 \end{array}$		  		  
Savage-Niehans Laplace	$a_1 \\ a_3$				$\checkmark$

No Rule fulfills all axioms  $\Rightarrow$  no perfect rule

Common usage: Remove duplicate Columns and use Laplace

Better: Define subjective probabilities and use them

# **Decision Graphs / Influence Diagrams**

## **Preference Orderings**

- A preference ordering ≽ is a ranking of all possible states of affairs (worlds) S
   these could be outcomes of actions, truth assignments, states in a search problem, etc.
  - $\circ$  s  $\succeq$  t: means that state s is at least as good as t
  - $\circ$   $s \succ t$ : means that state s is strictly preferred to t
- We insist that  $\succeq$  is
- $\circ$  reflexive: i.e., s  $\succeq$  s for all states s
- $\circ \$  transitive: i.e., if s  $\succeq$  t and t  $\succeq$  w, then s  $\succeq$  w
- $\circ$  connected: for all states s,t, either s  $\succeq$  t or t  $\succeq$  s

Structure of preference ordering imposes certain "rationality requirements" (it is a weak ordering)

- E.g., why transitivity?
  - Suppose you (strictly) prefer coffee to tea, tea to OJ, OJ to coffee
  - $\circ~$  If you prefer X to Y, you will trade me Y plus \$1 for X
  - I can construct a "money pump" and extract arbitrary amounts of money from you

## Utilities

Rather than just ranking outcomes, we must quantify our degree of preference  $\circ$  e.g., how much more important is *chc* than  $\sim mess$ 

A *utility function*  $U: S \to \mathbb{R}$  associates a realvalued *utility* with each outcome.  $\circ U(s)$  measures your *degree* of preference for s

Note: U induces a preference ordering  $\succeq_U$  over S defined as: s  $\succeq_U$  t iff  $U(s) \geq U(t)$ 

 $\circ$  obviously  $\succeq_U$  will be reflexive, transitive, connected

Under conditions of uncertainty, each decision d induces a distribution  $Pr_d$  over possible outcomes

•  $Pr_d(s)$  is probability of outcome s under decision d

The *expected utility* of decision d is defined

The *principle of maximum expected utility (MEU)* states that the optimal decision under conditions of uncertainty is that with the greatest expected utility.

$$EU(d) = \sum_{s \in S} Pr_d(s)U(s)$$

## **Decision Problems: Uncertainty**

A decision problem under uncertainty is: a set of decisions D

- $\circ\,$  a set of outcomes or states S
- an outcome function  $Pr: D \to \Delta(S)$ \*  $\Delta(S)$  is the set of distributions over S (e.g., Prd)
- $\circ$  a *utility function* U over S

A solution to a decision problem under uncertainty is any  $d^* \in D$  such that  $EU(d^*) \succeq EU(d)$  for all  $d \in D$ 

Again, for single-shot problems, this is trivial

# **Expected Utility: Notes**

Note that this viewpoint accounts for both:

- $\circ\,$  uncertainty in action outcomes
- uncertainty in state of knowledge
- $\circ\,$  any combination of the two



Stochastic actions

Rudolf Kruse, Matthias Steinbrecher, Pascal Held



Uncertain knowledge

Bayesian Networks

# **Expected Utility: Notes**

Why MEU? Where do utilities come from?

- $\circ~$  underlying foundations of utility theory tightly couple utility with action/choice
- a utility function can be determined by asking someone about their preferences for actions in specific scenarios (or "lotteries" over outcomes)

Utility functions needn't be unique

- if I multiply U by a positive constant, all decisions have same relative utility
- if I add a constant to U, same thing
- $\circ$  U is unique up to positive affine transformation

## So What are the Complications?

Outcome space is large

- $\circ\,$  like all of our problems, states spaces can be huge
- $\circ$  don't want to spell out distributions like  $Pr_d$  explicitly
- Solution: Bayes nets (or related: *influence diagrams*)

Decision space is large

- usually our decisions are not one-shot actions
- rather they involve sequential choices (like plans)
- if we treat each plan as a distinct decision, decision space is too large to handle directly
- Soln: use dynamic programming methods to *construct* optimal plans (actually generalizations of plans, called policies... like in game trees)

## So What are the Complications?

Decision networks (more commonly known as influence diagrams) provide a way of representing sequential decision problems
basic idea: represent the variables in the problem as you would in a BN

- add decision variables variables that you "control"
- add utility variables how good different states are

## **Sample Decision Network**



# **Decision Networks: Chance Nodes**

## Chance nodes

- $\circ\,$  random variables, denoted by circles
- $\circ\,$  as in a BN, probabilistic dependence on parents



## **Decision Networks: Decision Nodes**

## **Decision nodes**

- $\circ\,$  variables decision maker sets, denoted by squares
- parents reflect *information available* at time decision is to be made

In example decision node: the actual values of Ch and Fev will be observed before the decision to take test must be made

• agent can make different decisions for each instantiation of parents (i.e., policies)



# **Decision Networks: Decision Nodes**

## Value node

- $\circ\,$  specifies utility of a state, denoted by a diamond
- utility depends only on state of parents of value node
- $\circ\,$  generally: only one value node in a decision network

Utility depends only on disease and drug



## **Decision Networks: Assumptions**

Decision nodes are totally ordered  $\circ$  decision variables  $D_1, D_2, \ldots, D_n$ 

- decisions are made in sequence
- e.g., BloodTst (yes,no) decided before Drug (fd,md,no)

## No-forgetting property

 $\circ~$  any information available when decision  $D_i$  is made is available when decision  $D_j$  is made (for i < j)

• thus all parents of  $D_i$  are parents of  $D_j$ 



## Policies

Let  $Par(D_i)$  be the parents of decision node  $D_i$  $\circ Dom(Par(D_i))$  is the set of assignments to parents

A policy  $\delta$  is a set of mappings  $\delta_i$ , one for each decision node  $D_i$  $\circ \ \delta_i : Dom(Par(D_i)) \to (D_i)$ 

•  $\delta_i$  associates a decision with each parent assignment for  $D_i$ 

For example, a policy for BT might be:



Value of a policy  $\delta$  is the expected utility given that decision nodes are executed according to  $\delta$ 

Given associates  $\boldsymbol{x}$  to the set  $\boldsymbol{X}$  of all chance variables, let  $\delta(\boldsymbol{x})$  denote the assignment to decision variables dictated by  $\delta$  $\circ$  e.g., assigned to  $D_1$  determined by it's parents' assignment in  $\boldsymbol{x}$ 

 $\circ$  e.g., assigned to  $D_2$  determined by it's parents' assignment in  ${\boldsymbol x}$  along with whatever was assigned to D1

• etc.

Value of  $\delta$ :

$$EU(\delta) = \sum_{\boldsymbol{X}} P(\boldsymbol{X}, \delta(\boldsymbol{X}) U(\boldsymbol{X}, \delta(\boldsymbol{X}))$$

An optimal policy is a policy  $\delta^*$  such that  $EU(\delta^*) \ge EU(\delta)$  for all policies  $\delta$ 

We can use the dynamic programming principle yet again to avoid enumerating all policies

We can also use the structure of the decision network to use variable elimination to aid in the computation

## **Computing the Best Policy**

We can work backwards as follows

First compute optimal policy for Drug (last dec'n)

- $\circ\,$  for each assignment to parents (C,F,BT,TR) and for each decision value (D = md,fd,none), compute the expected value of choosing that value of D
- $\circ\,$  set policy choice for each value of parents to be the value of D that has max value



# **Computing the Best Policy**

Next compute policy for BT given policy  $\delta_D(C, F, BT, TR)$  just determined for Drug

- $\circ\,$  since  $\delta_D(C,F,BT,TR)$  is fixed, we can treat Drug as a normal random variable with deterministic probabilities
- $\circ\,$  i.e., for any instantiation of parents, value of Drug is fixed by policy  $\delta_D$
- this means we can solve for optimal policy for BT just as before
- only uninstantiated vars are random vars (once we fix *its* parents)

# **Computing the Best Policy**

How do we compute these expected values?

- $\circ\,$  suppose we have assigned < c, f, bt, pos > to parents of Drug
- we want to compute EU of deciding to set Drug = md
- we can run variable elimination!
- Treat C, F, BT, TR, Dr as evidence
  this reduces factors (e.g., U restricted to bt, md: depends on Dis)
  - eliminate remaining variables (e.g., only Disease left)
- $\circ~$  left with factor:  $U() = \sum_{Dis} P(Dis|c,f,bt,pos,md) U(Dis)$

We now know EU of doing Dr = md when c, f, bt, pos true

Can do same for fd, no to decide which is best



# **Computing Expected Utilities**

The preceding illustrates a general phenomenon

- $\circ\,$  computing expected utilities with BNs is quite easy
- utility nodes are just factors that can be dealt with using variable elimination

$$\begin{split} EU &= \sum_{A,B,C} P(A,B,C) U(B,C) \\ &= \sum_{A,B,C} P(C|B) P(B|A) P(A) U(B,C) \end{split}$$

Just eliminate variables in the usual way



If a decision node D has no decisions that follow it, we can find its policy by instantiating each of its parents and computing the expected utility of each decision for each parent instantiation

- no-forgetting means that all other decisions are instantiated (they must be parents)
- its easy to compute the expected utility using VE
- the number of computations is quite large: we run expected utility calculations (VE) for each parent instantiation together with each possible decision D might allow
- policy: choose max decision for each parent instant'n

# **Optimizing Policies: Key Points**

When a decision D node is optimized, it can be treated as a random variableo for each instantiation of its parents we now know what value the decision should take

• just treat policy as a new CPT: for a given parent instantiation  $\boldsymbol{x}$ , D gets  $\delta(\boldsymbol{x})$  with probability 1 (all other decisions get probability zero)

If we optimize from last decision to first, at each point we can optimize a specific decision by (a bunch of) simple VE calculations

• it's successor decisions (optimized) are just normal nodes in the BNs (with CPTs)

Decision networks commonly used by decision analysts to help structure decision problems

Much work put into computationally effective techniques to solve these

• common trick: replace the decision nodes with random variables at outset and solve a plain Bayes net (a subtle but useful transformation)

Complexity much greater than BN inference

- $\circ\,$  we need to solve a number of BN inference problems
- $\circ\,$  one BN problem for each setting of decision node parents and decision node value

## **Decision Network Notes**

In example on previous slide:

- we assume the state (of the variables at any stage) is fully observable
  \* hence all time t vars point to time t decision
- $\circ$  this means the state at time t d-separates the decision at time t-1 from the decision at time t-2
- so we ignore "no-forgetting" arcs between decisions
  - \* once you *know* the state at time t, what you *did* at time t-1 to get there is irrelevant to the decision at time t-1

If the state were not fully observable, we could not ignore the "no-forgetting" arcs

Setting: you want to buy a used car, but there's a good chance it is a "lemon" (i.e., prone to breakdown). Before deciding to buy it, you can take it to a mechanic for inspection. S/he will give you a report on the car, labelling it either "good" or "bad". A good report is positively correlated with the car being sound, while a bad report is positively correlated with the car being a lemon.

The report costs \$50 however. So you could risk it, and buy the car without the report.

Owning a sound car is better than having no car, which is better than owning a lemon.

## Car Buyer's Network



# Evaluate Last Decision: Buy (1)

$$\begin{split} EU(B|I,R) &= \sum_{L} P(L|I,R,B)U(L,B) \\ I &= i, R = g; \\ EU(buy) &= P(l|i,g)U(l,buy) + P(\sim l|i,g)U(\sim l,buy) - 50 \\ &= .18 \cdot -600 + .82 \cdot 1000 - 50 = 662 \\ EU(\sim buy) &= P(l|i,g)U(l,\sim buy) + P(\sim l|i,g)U(\sim l,\sim buy) - 50 \\ &= -300 - 50 = -350(-300 \text{ indep. of lemon}) \\ \text{So optimal } \delta_{Buy}(i,g) &= buy \end{split}$$

## Evaluate Last Decision: Buy (2)

$$I = i, R = b$$
:

$$\begin{split} EU(buy) &= P(l|i,b)U(l,buy) + P(\sim l|i,b)U(\sim l,buy) - 50 \\ &= .89 \cdot -600 + .11 \cdot 1000 - 50 = -474 \\ EU(\sim buy) &= P(l|i,b)U(l,\sim buy) + P(\sim l|i,b)U(\sim l,\sim buy) - 50 \\ &= -300 - 50 = -350(-300 \text{ indep. of lemon}) \\ \end{split}$$
 So optimal  $\delta_{Buy}(i,b) = \sim buy$ 

## Evaluate Last Decision: Buy (3)

 $I = \sim i, R = g$  (note: no inspection cost subtracted):

$$\begin{split} EU(buy) &= P(l|\sim i,g)U(l,buy) + P(\sim l|\sim i,g)U(\sim l,buy) \\ &= .5\cdot -600 + .5\cdot 1000 = 200 \\ EU(\sim buy) &= P(l|\sim i,g)U(l,\sim buy) + P(\sim l|\sim i,g)U(\sim l,\sim buy) - 50 \\ &= -300 - 50 = -350(-300 \text{ indep. of lemon}) \\ \text{So optimal } \delta_{Buy}(\sim i,g) = \sim buy \end{split}$$

So optimal policy for Buy is:  

$$\delta_{Buy}(i,g) = buy; \delta_{Buy}(i,b) = \sim buy; \delta_{Buy}(\sim i,n) = buy$$

Note: we don't bother computing policy for  $(i, \sim n)$ ,  $(\sim i, g)$ , or  $(\sim i, b)$ , since these occur with probability 0

## **Evaluate First Decision: Inspect**

$$EU(I) = \sum_{L,R} P(L, R|I) U(L, \boldsymbol{\delta_{Buy}}(I, R)),$$
  
where  $P(R, L|I) = P(R|L, I) P(L|I)$ 

$$EU(i) = .1 \cdot -600 + .4 \cdot -300 + .45 \cdot 1000 + .05 \cdot -300 - 50$$
  
= 237.5 - 50 = 187.5  
$$EU(\sim i) = P(l|\sim i, n)U(l, buy) + P(\sim l|\sim i, n)U(\sim l, buy)$$
  
= .5 \cdot -600 + .5 \cdot 1000 = 200

So optimal  $\delta_{Inspect}(\sim i) = buy$ 

	P(R,L I)	$\delta_{Buy}$	$U(L, \boldsymbol{\delta_{Buy}})$
g,l	0.1	buy	-600 - 50 = -650
$g, \sim l$	0.45	buy	1000 - 50 = 950
b, l	0.4	$\sim buy$	-300 - 50 = -350
$b, \sim l$	0.05	$\sim buy$	-300 - 50 = -350

# Value of Information

So optimal policy is: don't inspect, buy the car • EU = 200

- Notice that the EU of inspecting the car, then buying it iff you get a good report, is 237.5 less the cost of the inspection (50). So inspection not worth the improvement in EU.
- $\circ~$  But suppose inspection cost \$25: then it would be worth it (  $EU=237.5-25=212.5>EU(\sim i))$
- The *expected value of information* associated with inspection is 37.5 (it improves expected utility by this amount ignoring cost of inspection). How? Gives opportunity to change decision ( $\sim buy$  if bad).
- You should be willing to pay up to \$37.5 for the report

Slide of this section were taken from CSC 384 Lecture Slides ©2002-2003, C. Boutilier and P. Poupart