A FUZZY CONTROLLER FOR IDLE SPEED REGULATION

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Abstract

Fuzzy control is characterized by the treatment of vague control rules. For the serious application of such rule systems it is of major importance to clarify their semantic background. How essential these considerations are, is reflected by the fact that the way engineers use fuzzy control does often not coincide with the widespread understanding of control rules as logical statements or implications. Therefore fuzzy control should rather be seen as an interpolation of a partially specified control function in a vague environment, reflecting the indistinguishability of measurements or control values.

In this paper we outline that the concept of equality relations is a natural way to represent such vague environments. Furthermore we show how our resulting view of fuzzy control has successfully been applied to develop a well-founded (generalized) fuzzy controller for idle speed regulation of a car engine.

1 Introduction

In the field of approximate reasoning and knowledge based systems we are often concerned with the problem of handling linguistic expert rules of the form

$$(*)\ R_r: \quad \textit{if } \xi_1 \textit{ is } A_r^{(1)} \textit{ and } \ldots \textit{ and } \xi_n \textit{ is } A_r^{(n)} \textit{ then } \eta \textit{ is } B_r,\ r = 1, \ldots, k,$$

where $A_r^{(i)}$ and B_r denote linguistic terms that are interpreted by fuzzy sets $\mu_r^{(i)} : X_i \to [0,1]$ and $\nu_r : Y \to [0,1]$ w.r.t. underlying domains $X_1, X_2, \ldots, X_n$, and Y, respectively, on which the variables $\xi_1, \ldots, \xi_n$, and η can take their values [3]. The aim of this paper is to present one approach to a well-founded semantics for such approximate rules.

In order to achieve an acceptable interpretation, the following three problems have to be solved:

- What is the meaning of the involved fuzzy sets ?
- Based on this meaning and consistent with it, what is the semantics of a single approximate rule?
- Finally, what is the semantics of the whole system of rules?

In Section 2 we will answer these questions from a *physical* point of view, where *fuzzy sets* (vague objects) are used, considered as representatives of crisp data in a vague environment, and formalized by the concept of an *equality relation* [5, 7].

As one result it turns out that the well-known (heuristic) inference mechanism of *Mamdani's fuzzy controller* [12] can be justified based on the corresponding mathematical structures. Furthermore the methodology introduced in this paper has been applied successfully in a case study of idle speed control for the Volkswagen Golf GTI. Section 3 outlines the underlying control problem, the realization of the controller, and its comparison with the existing production-line controller.

2 Fuzzy Rules and Equality Relations

Our first approach to understand the meaning of approximate rules of the type $(*)$ is based on the idea that each fuzzy set which corresponds to a linguistic term is a representative of a concrete crisp value. But according to an existing vague environment, the handling of this crisp value is only admissible in the more vague form of a fuzzy set. In order to elucidate these concepts, we will motivate and explain the notion of a vague environment.

Vague Environments

Let us consider the following situation, which appears in many problems of physics and engineering. We are interested in a variable ξ that can take values on the real interval $[a, b]$. If we want to measure ξ, we have to take into account that our measuring instrument is not able to provide a value for ξ with 100% exactness. Trying to adjust ξ instead of measuring it, we have to bear in mind that our instruments will never regulate ξ in the way that we obtain exactly the desired value. The reasons for this imprecision might be found in the inappropriateness of our instruments. But in many cases also the experiment or the environment make it impossible to get exact values. For example, it does not make sense to speak of a room temperature of 21.0236507 °C. In this case it is even not of any interest to reach arbitrary exactness.

We summarize that we have to deal with enforced inexactness (according to inappropriate instruments or a 'bad' environment) and with intended inexactness (f.e. room temperature). Both types of inexactness involve the following problem.

In order to represent the inexactness often a (small) number $\epsilon > 0$ is chosen. Then two values $x, x' \in [a, b]$ are identified if $|x - x'| < \epsilon$ holds. Mathematically, we can represent this notion by a relation $R_\epsilon \subseteq [a, b] \times [a, b]$ containing all pairs (x, x') satisfying $|x - x'| < \epsilon$, i.e.

$$R_\epsilon = \{(x, x') \in [a, b] \times [a, b] \mid |x - x'| < \epsilon\}$$

This relation is reflexive (x is identified with x; $|x - x'| < \epsilon$) and symmetric (if x is identified with x', then x' is also identified with x; $|x - x'| < \epsilon \Rightarrow |x' - x| < \epsilon$), but not transitive. The non-transitivity of this relation means that, although we identify x with x' (i.e. $|x - x'| < \epsilon$) and x' with x'' (i.e. $|x' -$

$x''| < \epsilon$), we might not necessarily identify x with x'' (i.e. $|x - x''| > \epsilon$ is possible). This situation is also known as the Poincaré paradox [4].
The consequence of the non-transitivity is, that we cannot introduce equivalence classes induced by the relation R_ϵ. A discretization of the set $[a, b]$ corresponds to choosing values $x_1, \ldots, x_n \in [a, b]$ as representatives. Values $x \notin \{x_1, \ldots, x_n\}$ are identified with one of the values $x_1, \ldots, x_n$, generally with the value x_i for which $|x - x_i| < \epsilon$ holds. The class of values x identified with the representative x_i strongly depends on the choice of x_i according to the non-transitivity of R_ϵ. In this sense we can say that the relation R_ϵ contains more information than the discretization $\{x_1, \ldots, x_n\}$ with its classes.
In many cases we are not only considering one acceptability bound ϵ for the decision if two values should be identified or not. We are now interested in a set of such acceptability bounds. Without loss of generality, it is sufficient to consider all $0 < \epsilon \leq 1$. (Instead of, f.e., looking at all $0 < \epsilon \leq 2$ one could measure the distance $|x - x'|$ between two points x and x' by $\frac{1}{2}|x - x'|$, leading again to $0 < \epsilon \leq 1$.) This means, we have to take into account all relations R_ϵ for $0 < \epsilon \leq 1$.
We obviously have $R_\epsilon \subseteq R_{\epsilon'}$ for $\epsilon \leq \epsilon'$.
Therefore it is sufficient to know the value

$$\begin{aligned} R(x, x') &= \sup \{\epsilon \in [0, 1] \mid (x, x') \in R_\epsilon\} \\ &= \min \{|x - x'|, 1\} \end{aligned}$$

in order to decide whether $(x, x') \in R_\epsilon$ holds or not.
The greater $R(x, x')$ is, the less we can identify x and x'. If we consider the value $E(x, x') = 1 - R(x, x') = 1 - \min\{|x - x'|, 1\}$, we can say that, the greater $E(x, x')$ is, the more x and x' can be identified.
Of course, at first sight it seems artificially complicated to consider $E(x, x')$ instead of $1 - \min\{|x - x'|, 1\}$ directly. But this holds only for the simple case where we measure the distance between two values x and x' by the standard metric $|x - x'|$. In many applications, the standard metric is not an appropriate measure. For example, in the case of room temperatures, it does not make a big difference whether the temperature is 32 °C or 36 °C (Both temperatures are much too high). But the difference between 21 °C and 25 °C is really considerable. For this problem we would need a non-linear transformation of the set of possible room temperatures in order to reflect the notion, that we do not distinguish well between high temperatures, but we do make finer differences between temperatures in the normal range. In such cases it might be more convenient to consider $E(x, x')$ instead of trying to find a corresponding non-linear transformation.
Mathematically, if X is the set of possible values, then E is a mapping from $X \times X$ to the unit interval. It is easy to verify that E should satisfy the following axioms

$$\begin{array}{lll} (i) & E(x, x) = 1 & \text{(total existence)} \\ (ii) & E(x, x') = E(x', x) & \text{(symmetry)} \\ (iii) & E(x, x') + E(x', x'') - 1 \leq E(x, x'') & \text{(transitivity)} \end{array}$$

Such a mapping is called an *equality relation* on X [5, 7] or similarity relation [14]. Indeed, in the above sense $E(x, x')$ can be interpreted as the degree to which x and x' can be identified or to which x and x' are equal.
From a logical viewpoint condition (i) corresponds the axiom $x = x$, whereas (ii) can be interpreted as $x = x' \rightarrow x' = x$. The logical axiom for transitivity is $x = x' \wedge x' = x'' \rightarrow x = x''$. Valuating this formula using the t-norm T, where $T(\alpha, \beta) = \max\{\alpha + \beta - 1, 0\}$, translates it to the condition (iii). The use of the t-norm associated with Lukasiewicz Logic in this context is induced by the triangular inequality for metrics. Of course, in general in connection with equality relations also other t-norms are considered.
If we consider a crisp value $x \in X$ in a vague environment (i.e. there is an equality relation E on X), it is reasonable to 'add' to x_0 all those values $x' \in X$ which are equal to some degree to x_0, in the same way as we add all elements equivalent to x_0 when we consider equivalence classes. This means, instead of the set $\{x_0\}$, we should consider the fuzzy set $\mu_{x_0} : X \rightarrow [0, 1]$, $\mu_{x_0}(x) = E(x_0, x)$. In this sense, such a fuzzy set can be understood as the representation of a crisp value in a vague environment.
These considerations lead us to the idea that fuzzy sets might be interpreted in the above sense, even if the equality relation E is not explicitly given or if it is unknown.
The problem of deriving an appropriate equality relation from given fuzzy sets is solved in [6, 10, 11], and for reasons of simplicity, we will not consider this problem in this paper. But we emphasize the fact that the results of the above mentioned papers indicate the opportunity of interpreting fuzzy sets as crisp values in vague environments in many cases.
Note that the mapping E is sometimes also called indistinguishability operator [15, 8]. But the corresponding approaches do not specify concrete semantics, such as we have provided in this section, where equality relations are considered as representations of indistinguishabilities w.r.t. a set of tolerance bounds. These bounds are directly connected with the physical environment.
Another class of approaches to the interpretation of approximate rules is based on deduction in logical systems as, for example, possibilistic logic [3] or fuzzy logic as described in [13]. These approaches are designed to handle logical formulae, to which numerical weights are assigned. In opposition to equality relations, where a concrete universe of discourse is given, logical calculi are more appropriate for coping with axiomatic descriptions of situations.

Interpretation of a Single Rule

We now consider the problem of deriving information about the value for η given $\xi_1 = x_1, \ldots, \xi_n = x_n$ and one rule of the form (*).
We assume that there are equality relations $E_1, \ldots, E_n, F$ on $X_1, \ldots, X_n, Y$, respectively, such that the fuzzy sets involved in the rule can be seen as representatives of crisp values in the corresponding vague environments. The equality relations might be given explicitly or they can be derived as described in [6, 10]. We also assume, that the fuzzy set $\mu_i^{(r)}$ $(i = 1, \ldots, n)$, $\mu^{(r)}$, corresponding to the linguistic term $A_i^{(r)}, B_i^{(r)}$, respectively, represents the value $x_i^{(r)}, y^{(r)}$ in the respective vague environment. Therefore, we consider fuzzy sets as induced concepts, i.e. as representations of crisp values in vague environment characterized by an equality relation. Thus we assume a very restrictive, but clear interpretation of fuzzy sets. We do not claim that this interpretation is the only one. However, it seems to be well suited for control applications.

The Relational Interpretation

In order to apply the rule we consider the product space $X = X_1 \times \cdots \times X_n \times Y$. Since the sets $X_1, \ldots, X_n, Y$ can be viewed as vague environments according to the equality relations, X should also be considered as a vague environment, i.e. we have to define an equality relation E on this set. The coarsest (greatest) equality relation E on X, satisfying

$$\sup_{\substack{(x_1,\ldots,x_{i-1},x_{i+1},\ldots,x_n,y),\\(x'_1,\ldots,x'_{i-1},x'_{i+1},\ldots,x'_n,y')\in X_1\times\cdots\times X_{i-1}\times X_{i+1}\times\cdots\times X_n\times Y}} E((x_1,\ldots,x_n,y),(x'_1,\ldots,x'_n,y')) = E_i(x_i,x'_i) \qquad (\text{for } i=1,\ldots,n)$$

and

$$\sup_{\substack{(x_1,\ldots,x_n),\\(x'_1,\ldots,x'_n)\in X_1\times\cdots\times X_n}} E((x_1,\ldots,x_n,y),(x'_1,\ldots,x'_n,y')) = F(y,y')$$

is given by

$$E((x_1,\ldots,x_n,y),(x'_1,\ldots,x'_n,y')) = \min\{E_1(x_1,x'_1),\ldots,E_n(x_n,x'_n),F(y,y')\}.$$

E reflects the independence of the equality relations $E_1,\ldots,E_n$ and F. For a more detailed justification of the definition of E see [9, 10].

The rule $R^{(r)}$ represents the point $(x_1^{(r)},\ldots,x_n^{(r)},y^{(r)}) \in X$. Since we are given fixed inputs $x_1,\ldots,x_n$, we obtain for each $y \in Y$ the degree to which $(x_1^{(r)},\ldots,x_n^{(r)},y^{(r)})$ and $(x_1,\ldots,x_n,y)$ can be identified. This degree can be computed by

$$E((x_1^{(r)},\ldots,x_n^{(r)},y^{(r)}),(x_1,\ldots,x_n,y)) = \min\{\mu_1^{(r)}(x_1),\ldots,\mu_n^{(r)}(x_n),\mu^{(r)}(y)\}$$

Thus we obtain a fuzzy set on Y of possible values for η. This fuzzy set is equal to the fuzzy set, derived from one rule of the Mamdani–Controller [12].

The Functional Interpretation

For this interpretation, we assume that the $R^{(r)}$ correspond to the notion that we have to map the input $(x_1^{(r)},\ldots,x_n^{(r)})$ to the output-value $y^{(r)}$. We also assume that if $(x_1,\ldots,x_n)$ and $(x'_1,\ldots,x'_n)$ can be identified to a degree of at least ϵ, then the corresponding output-values y and y' should also be identified to a degree of at least ϵ. In this case, for the application of the rule $R^{(r)}$, we would have to check, to which degree $(x_1,\ldots,x_n)$ and $(x_1^{(r)},\ldots,x_n^{(r)})$ can be identified, i.e. we have to compute the value

$$\min\{\mu_1^{(r)}(x_1),\ldots,\mu_n^{(r)}(x_n)\}.$$

As possible output-values we obtain the crisp set

$$(**)\ \left\{y \in Y \;\middle|\; \mu^{(r)}(y) \geq \min\{\mu_1^{(r)}(x_1),\ldots,\mu_n^{(r)}(x_n)\}\right\}$$

Interpretation of a Set of Rules

After we have given two possible interpretations for the application of a single rule, we now explain how the results of a set of rules have to be combined.

In the relational interpretation, each rule $R^{(r)}$ leads to an output-fuzzy set that reflects the idea that we identify the input $(x_1,\ldots,x_n)$ with the tuple $(x_1^{(r)},\ldots,x_n^{(r)})$. We therefore have to consider the union of all these fuzzy sets, i.e. the fuzzy set on Y which yields the membership degree

$$\max_{r=1,\ldots,n}\left\{\min\{\mu_1^{(r)}(x_1),\ldots,\mu_n^{(r)}(x_n),\mu^{(r)}(y)\}\right\}$$

for $y \in Y$. This is exactly the output-fuzzy set of the Mamdani-Controller [12].

In the functional interpretation, each rule $R^{(r)}$ enforces a restriction in the form of the set $(**)$ for the possible values for y. Since all these restrictions have to be satisfied, we obtain the intersection of the sets $(**)$ as the set of possible values for y.

3 Application to Idle Speed Control

In the previous section we introduced a theoretical and semantic approach to fuzzy control. It has to be checked, whether the basic concept of equality relations and the presented results are appropriate for solving existing control problems of industrial practice.

For this reason in cooperation between Volkswagen AG, Wolfsburg, Germany, and the Department of Computer Science of the University of Braunschweig a idle speed controller for the Volkswagen 2000cc 116hp petrol engine of the Golf GTI model has been developed.

The controller is based on a cognitive analysis of the underlying motor process and consists of a Mamdani–Controller which is embedded in a so-called "meta-controller". It turns out that the resulting fuzzy controller has a better performance than the corresponding production-line controller. Therefore our methods will also be applied to idle speed control problems with respect to other petrol engines of Volkswagen AG and Audi AG, respectively.

Note that from an engineer's point of view fuzzy control refers to interpolation of a partially specified control function in a vague environment. For this reason the whole application is focused on the *functional* interpretation of control rules as discussed in Section 2 In practice it turned out that engineers are thoroughly in the position to define appropriate equality relations based on available information about existing quantitative dependencies between the considered input and output variables.

A Short Introduction to Idle Speed Control

Nowadays the intended performance standards of petrol engines make it necessary to decrease the consumption of petrol and the emission of toxic agents. One specific problem in this field is the reduction of idle speed, since the enlarged application of comfort facilities like, for example, air-conditioning systems or power steerings, require a flexible control mechanism, as the exalted load leads to heavy drops of revolution.

Generally there are two different kinds of idle speed control:

- Ignition Adjustment

- Charge Control

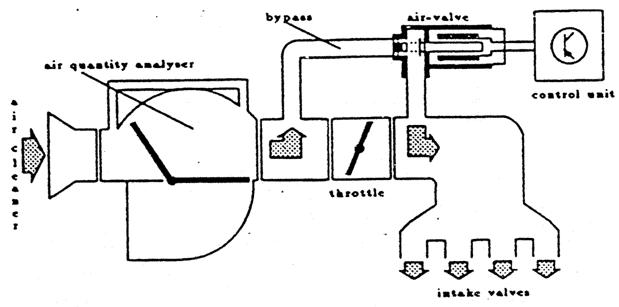

Figure 1: *Principle of idle speed control*

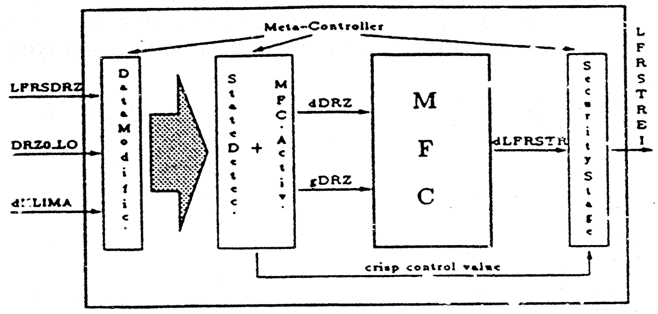

Figure 2: *Construction of the fuzzy controller*

In order to be most effective, our fuzzy controller only realizes charge control. The ignition adjustment of the production-line car is retained.

The principle of charge control is shown in Figure 1. The air-valve resides in a bypass to the throttle, and varies, if required, the bypass cross-section.

On the existing vehicle a sudden drop of revolution at idling may have various reasons [1, 2]:

- switching-on electrical units, which load the engine through the three-phase generator
- switching-on the air-conditioning system, which load the engine through the air-conditioning compressor (including the additional cooling fan)
- activation of power steering, where the hydraulic circuit pump loads the engine

The task of charge control is to compensate the drop of revolution by enlarging the cross-section. In this case the number or revolutions should get the target rotation speed as correct and as fast as possible.

One of the main problems is the low torque in range of low revolutions, because in extreme situations, like simultaneous switching-on of the air-conditioning system and the power steering, very strong and rapidly occurring drops of revolution are the consequences.

The above-mentioned lacking quality of engine speed information refers to the imprecision of the available signal of the Hall-pick-up in the ignition distributor (i.e. differences in the number of revolutions up to ± 30 rpm result from manufacturing tolerances, gear clearances, and torsional vibrations).

Another problem arises from the plurality of additional stochastic processes in the system. As an example, bad combustions or deviations of ignition and fuel-injection have to be mentioned, because in this case a charge controller should not react on a differing engine speed in order to prevent an oscillation of the idle speed.

However, the delay of the automatic control system turns out to be the hardest problem, since it passes about ten sequences of ignition after an alteration of charge, until the engine delivers the scaled torque (essential reason: term of airflow).

Design of the Cognitive Controller

The developed fuzzy controller presents an integrated conception of two controllers to solve the problems mentioned in Section 3 A Mamdani-Controller (MFC) forms the basic unit, which is embedded in a so-called 'meta-controller' (Figure 2).

The deviation dDRZ of the engine speed to the target rotation speed and the gradient gDRZ of the deviation are the two inputs of the MFC. The change of current dLFRSTR for the air-valve is the output.

The MFC was realized by application of equality relations, based on the modelling of vague environments that follows the principle of indistinguishability, as discussed in Section 2. The fuzzy sets induced by the equality relation E_{gDRZ} with their associated linguistic terms like *nb* or *ps*, which are needed only for the denomination, are shown in Figure 3. Analogously, we denote the corresponding associated linguistic terms for gDRZ and dLFRSTR, respectively.

The 49 tuples specified by the expert, and the three equality relations E_{dDRZ}, E_{gDRZ} and $E_{dLFRSTR}$ constitute the rule base shown in Figure 4.

For defuzzification, we used the center-of-area method. Figure 5 illustrates the induced look-up table of the MFC.

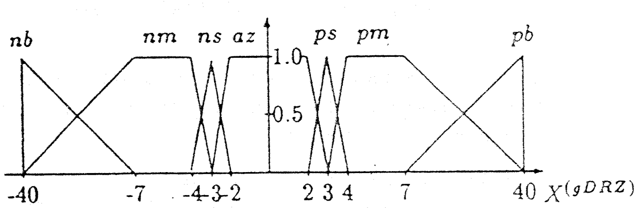

Figure 3: : *The fuzzy partition of gDRZ*

gDRZ

dDRZ	NB	NM	NS	ZR	PS	PM	PB
NB	PH	PB	PB	PM	PM	PS	PS
NM	PH	PB	PM	PM	PS	PS	ZR
NS	PB	PM	PS	PS	ZR	ZR	ZR
ZR	PS	PS	ZR	ZR	ZR	NM	NS
PS	ZR	ZR	ZR	NS	NS	NM	NB
PM	ZR	NS	NS	NM	NB	NB	NH
PB	NS	NS	NM	NB	NB	NB	NB

Figure 4: *Rulebase*

The fuzzy controller is implemented on a 386-processor laptop (program language: C). The data communication takes place through the control unit of the engine management system. The inputs are the number of revolutions DRZ0_LO, the target rotation speed LFRSDRZ, and the state flag dKLIMA of the air-conditioning system (on/off). The value of current LFRSTREI for the air-valve serves as output. The meta–controller consists of three components: data modification, state detection (including MFC activation), and security stage. The data modification computes averages from the original data and supplies evaluatable engine speed information. If required, the state detection activates the MFC, and a new control value is determined using the modified and the original data. A security stage behind the MFC takes care of the control limitation. At the time of switching on the air-conditioning system, the MFC is not activated, and a fixed crisp output value has to be chosen to get the fastest control action.

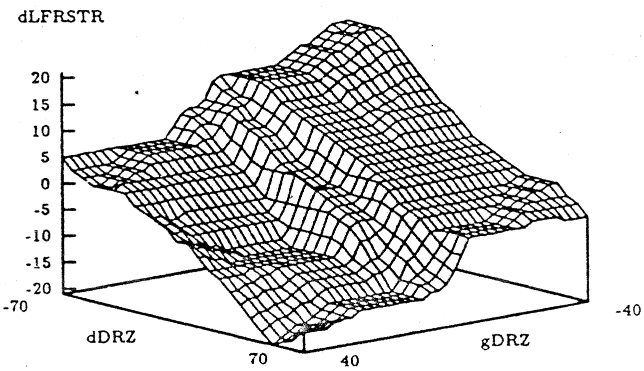

Figure 5: *Look-up table of the MFC*

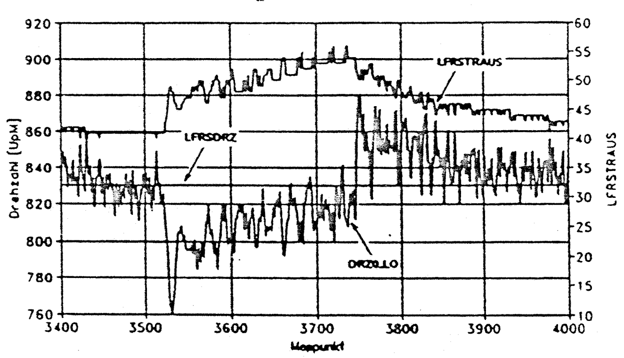

Figure 6: *Power steering with prod. control*

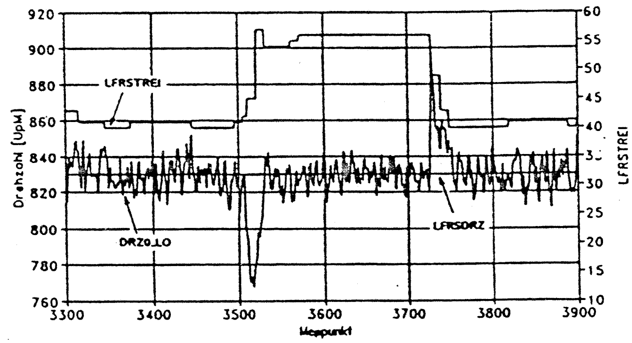

Figure 7: *Power steering with cogn. control*

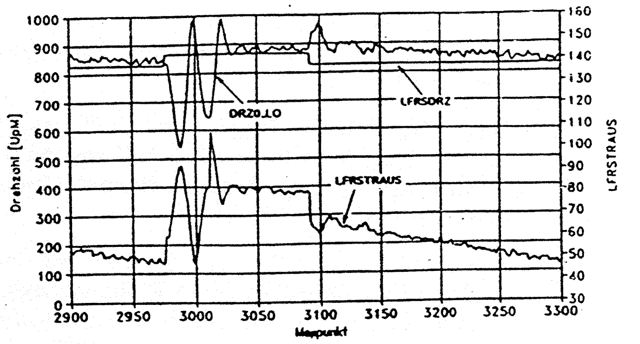

Figure 8: *Air–cond. syst. with prod. control*

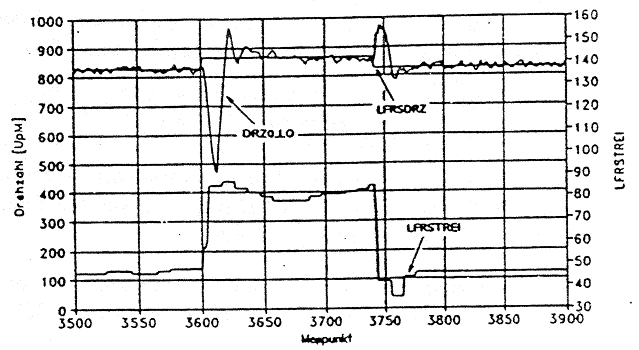

Figure 9: *Air–cond. syst. with cogn. control*

Comparison to the Production–Line Controller

Figures 6 to 9 show a comparison of the cognitive controller and the production–line controller with respect to two kinds of load. First we have activated the power steering about six seconds, and then, for about three seconds, we have switched on the air–conditioning system. A measuring point complies to one sequence of ignition (i.e. one stroke or the time for 180°angle of crank shaft). LFRSTRAUS and LFRSTREI illustrate in each case the time–dependent output of the production–line controller and the cognitive controller, respectively. Figures 6 and 7 show the situation during the activation of power steering. Figures 8 and 9 depict the performance by switching on the air–conditioning system. While switching an additional load, the over– and under–vibration of engine speed is not controllable physically because of the delay.

In comparison to the production-line controller, the developed cognitive controller possesses a quiet smooth control characteristic. In addition, we recognized a fast and precise reach of the target rotation speed and also a great stability on slowly increasing load. Moreover, there are no vibrations after extreme alterations of load. The cognitive approach allows a good state parting, a simple voting treatment, and a short period of development.

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